MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Sir FRANK DYSON, M.A., LL.D., F.R.S., VICE-PRESIDENT, in the Chair.

George Beaver Croucher, F.R.G.S., Principal Grimsby Navigation School and Technical School for Fishermen, Grimsby;
Thomas Henry Dean, Edenthorpe, Anderton Road, Sparkbrook, Birmingham;
Algernon Charles Gifford, M.A., Wellington College, Wellington, New Zealand;
John G. Howarth, B.A., Science Master of Reigate Grammar School, Dane's Dyke, Reigate, Surrey;
Harold Jeffreys, B.A., M.Sc., Isaac Newton Student, St. John's College, Cambridge;
James Knight, M.A., D.Sc., F.R.S.E., F.C.S., F.G.S., Royal Philosophical Society, Glasgow;
John Earle Maxwell, 84 Dartmouth Road, Cricklewood, N.W.;
Clive Nossiter, Acting First Assistant, Perth Observatory, Western Australia; and
William Marshall Smart, B.A., B.Sc., Trinity College, Cambridge,

were balloted for and duly elected Fellows of the Society.

The following candidate was proposed for election as a Fellow of the Society, the name of the proposer from personal knowledge being appended:—

William Blastall Wilkinson, Lieut. R.N.R., H.M.S. *Encounter*, Sydney, New South Wales, Australia (proposed by E. B. Simpson-Baikie).

Thirty-one presents were announced as having been received since the last Meeting, including, amongst others:—

Regimento do Estrolabio e do Quadrante: Tractado da Spera do Mundow. Reproduction facsimile, presented by the Portuguese Minister of Public Instruction.

The Total Eclipse of the Sun, 1914 *August* 21. By the Secretary, Joint Permanent Eclipse Committee.

(*Communicated by the Joint Permanent Eclipse Committee.*)

The central line of the Total Eclipse of 1914 August 21 extended from Greenland through Norway, Sweden, and Russia, to Persia. In Sweden and Russia the duration of totality was a little over two minutes, and meteorological data indicated reasonable chances of fine weather on the day of eclipse, especially in South Russia.

In view of the convenient location of this eclipse, and in consideration of the numerous problems which can only be attacked on such occasions, the Joint Permanent Eclipse Committee was of opinion that at least three stations should be occupied. The Astronomer Royal and Professor Newall early announced their intention of sending out expeditions from the Royal Observatory, Greenwich, and the Solar Physics Observatory, Cambridge, respectively, and it was resolved that these should be supplemented, if possible, by an expedition under the auspices of the Committee, with the aid of the Government Grant Fund.

After consultation with Dr. Backlund, the Imperial Astronomer for Russia, it was arranged that the Committee's expedition should be located at Kiev, the Greenwich party at Minsk, and the Cambridge party at Theodosia in the Crimea. It was originally intended that the Kiev observers should be Professor Fowler and Mr. W. E. Curtis of the Imperial College, South Kensington, with Major Hills as volunteer observer, for one of the series of observations; the Rev. Father Cortie, with Rev. Father O'Connor of Stonyhurst College as volunteer observer, for another series. Eventually, however, difficulties arose in connection with the admission of Fathers Cortie and O'Connor into Russian territory, and it became necessary to modify the original plans. The Stonyhurst party accordingly went to Hernösand in Sweden.

The eclipse was successfully observed at Minsk and Hernösand, but observations at Theodosia were unfortunately prevented by clouds. The Kiev party did not leave England so early as the others, and difficulties arising from the outbreak of war led to the abandonment of the expedition.

The thanks of the Committee are due to Dr. Backlund and the Russian Imperial Government for the special facilities granted

to observers going to Russia; instruments were admitted duty free, and arrangements were made for their free conveyance over the State railways; observers were further permitted to travel by rail at half the ordinary fares during a period of two months.

Thanks are also due to Dr. Cowell, Superintendent of the *Nautical Almanac*, for calculations of the circumstances of the eclipse at selected stations, issued as *Nautical Almanac Circular*, No. 20.

The Committee is especially indebted to the Government Grant Committee for funds. Apart from the personal expenses of Major Hills and Father O'Connor, the grant provided for the expenses of the expeditions to Kiev and Hernösand, including adaptation of some of the instruments, and also for the expenses of one of the observers attached to the expedition to Theodosia.

Other assistance is acknowledged in the reports on the plans and work of the observing parties.

Preliminary Report on the Total Solar Eclipse of 1914 *August* 21. (*Observed by the Expedition of the Joint Permanent Eclipse Committee to Hernösand, Sweden.*)

By the Rev. A. L. Cortie, S.J., F.R.A.S. (Plates 9–12.)

(*Communicated by the Joint Permanent Eclipse Committee.*)

1. *General Arrangements.*

The expedition was one of those organised by the Joint Permanent Eclipse Committee of the Royal Society and of the Royal Astronomical Society, the expense of the expedition being defrayed by a grant made by the Government Grant Committee.

It had been arranged that Father Cortie, with Father O'Connor as a volunteer observer, should form part of the expedition to Kiev with Professor Fowler, Major Hills, and Mr. Curtis. But owing to the refusal of the Russian Government, communicated by the Foreign Office, to allow the two Jesuit Fathers to enter the Empire, it was decided to divide the expedition originally intended for Kiev, and that Father Cortie should lead an expedition to Hernösand, Sweden. Since our return we have learned that Dr. Backlund, the Imperial Astronomer at Pulkovo, obtained permission for us to go to Russia—a permission which fortunately, as events proved, was not communicated to us. Hernösand had not been chosen as a station for observation of the eclipse in the original scheme of the British expeditions, on account of the average weather conditions in August not being so favourable for clear skies as at the stations in Russia. As has not unfrequently happened, prognostications founded on meteorological results were of very little use as a guide in the selection of the eclipse stations.

Besides Father Cortie, the eclipse party consisted of Father O'Connor of the Stonyhurst College Observatory, Mr. G. J. Gibbs of Preston, and Mr. E. T. Whitelow of Birkdale, Southport. We left Hull by the s.s. *Calypso* on July 28, and arrived at Gothenburg on the morning of July 30, and thence by rail to Stockholm, arriving the evening of the same day. The time had been short for making preliminary arrangements as to a possible site for the instruments at Hernösand. It happened fortunately that Father Wulf and his assistant Father Rodés of St. Ignatius College, Valkenburg, in Holland, had arranged to observe the eclipse at this station, and through their kind offices we were put into communication with Professor B. Hasselberg of the Academy of Sciences, Stockholm, the chief of the Swedish Commission for the observation of the eclipse. We owe it to his great kindness and courtesy that we were enabled to secure an excellent site at the Technical School at Hernösand, and that our instrumental equipment was admitted duty-free into Sweden, the officials of the Custom-house having also received instructions to expedite their despatch in every possible way. We left Stockholm by boat on August 2, and arrived at our destination on August 3. We found our instruments, which had been sent from Hull in the middle of July, safely stored in the Custom-house. They were delivered next morning and transferred to the Technical School. After a merely formal examination by a Customs official, we immediately commenced with their erection. Everyone was exceedingly kind and most anxious to do all in their power to assist us. As war had been declared, and the Swedes, though neutral, had mobilised, the only drawback on our perfect liberty was that, in common with all other strangers in the town, we had to present ourselves every day of our stay to the police.

2. *Erection of Instruments.*

We cannot be too grateful to Herr Recktor Tham for placing the Technical School practically at the entire disposal of the British party and of that from Valkenburg for the observation of the eclipse. The school contained well-appointed laboratories, mechanic's and carpenter's shops, dark room, and a complete electrical plant in connection with the town mains; in fact, every convenience an eclipse party could desire. The town of Hernösand is situated partly on the mainland and partly on an island, the two portions of the town being separated by a strait on which is the port, the strait being spanned by two bridges. The Technical School is on the island, and its co-ordinates, as given me very kindly by Professor B. Hasselberg, are:—Latitude 62° 38′ 8·8″ N., and longitude 17° 57′ 32·25″ E. of Greenwich. The field at the back of the school in which we elected to set up our instruments has a gradual rising slope, and in addition the school itself stands on rising ground, so that the altitude of the station was about 300 feet above sea-level. The meridians and azimuths of the

and dark sky was superb. Such views as we could take in the intervals of exposures showed a long fish-tail streamer on the western side of the sun, a brilliant streamer N.E., a shorter and less brilliant streamer S.E., and between them a long equatorial ray, but much fainter than either. No one seems to have seen the prominence in the S.E. quadrant which is such a marked feature on the photographic plates. Nor were the polar rays very marked visually. The general impression from visual observations was that the west side of the corona was of the minimum type, while the east side was of an intermediate type when the streamers begin to open out polewards. During totality the planet Mercury shone brilliantly to the N.W. of the sun, and the planet Venus low down on the S.E. The eclipse was a very bright one. About 20 minutes before totality a decidedly cold wind blew from the S.E. The total drop in temperature during the eclipse was 7°·4F.

The plates were all developed and the instruments packed on August 21 and 22. We took the first boat possible from Hernösand to Stockholm, which, however, did not leave until August 25; and, passing safely through two Swedish mine-fields between Sundsval and Stockholm, under the escort of armed vessels, we arrived safely at the Swedish capital on the afternoon of August 27. There his Excellency the British Minister, Sir Esmé Howard, very kindly took charge of our instruments on account of the war, and subsequently transmitted them to the British Vice-Consul at Bergen, who secured their safe transit across the North Sea to Hull. We left Stockholm on August 28 by rail for Bergen, whence we started next day on board a Norwegian steamer for Newcastle. Warned twice by British cruisers, which were patrolling the North Sea, we were enabled to avoid a German mine-field some distance from the mouth of the Tyne, by making for the north of Scotland and coming down the coast. Finally we entered the Tyne on the early morning of August 31, escorted by torpedo boats.

4. *Summary of Results.*

Five large-scale photographs of the corona were secured which show much fine detail. Also four photographs were taken with the Abney coronagraph, and one with the Ross lens, by Mr. Whitelow, which show the extension of the coronal streamers further than was observed with the naked eye by the Swedish students who made the composite drawing on the graduated chart. The drawing is in excellent accord with the main features as photographed; but whereas the greatest extent of the corona on the drawing is $1\frac{1}{8}$ lunar diameter, the 10-seconds photograph taken with the Abney coronagraph shows the extension in the S.E. streamer to have been fully 2 diameters. Both photograph and drawing agree in showing a double curvature in this streamer. It first bends upward, and then downward, until it terminates almost in a point.

The eclipse is interesting as having occurred at a period when

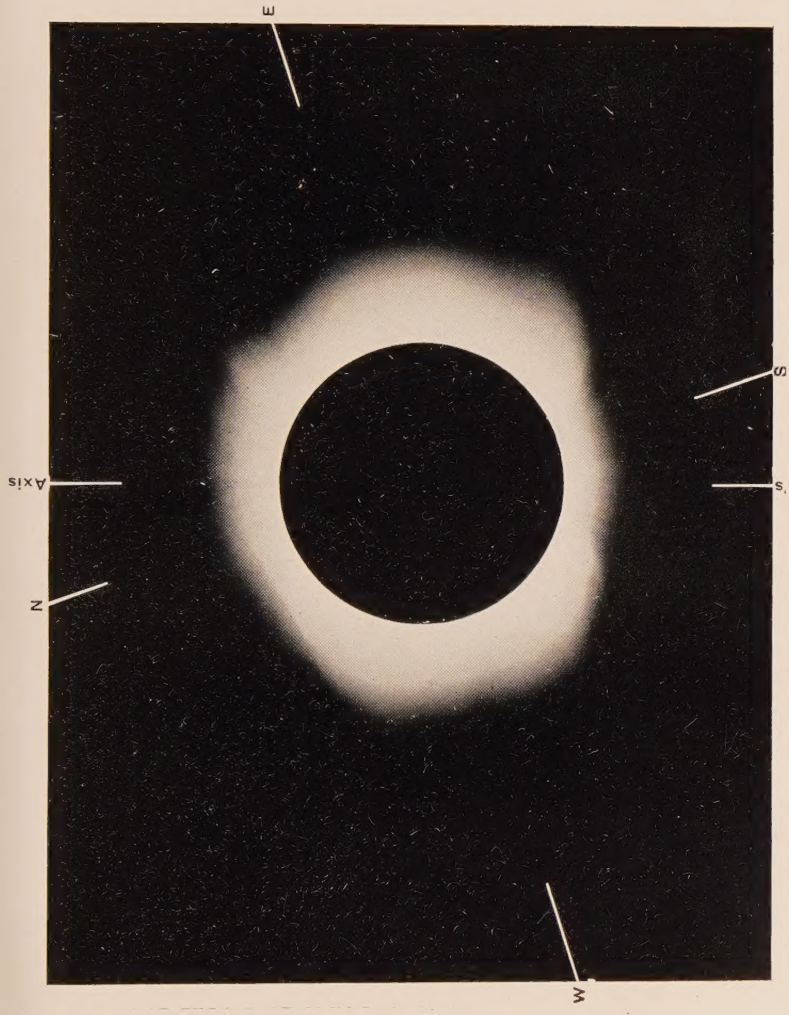
the solar-spot activity was increasing; very few, if any, former coronas having been photographed at a similar phase of the sun-spot cycle. The preceding minimum of spot-activity had also been very protracted. The general form of the corona corresponds neither to a "maximum" nor a "minimum" corona. The long fish-tail streamer on the west side, the roots of which extend from position-angle 230° to 295°, combined with the polar rays, which in the N. cover an arc of 75°, between position-angles 330° and 45°, and on the S. an arc of 65°, between position-angles 160° and 225°, are features characteristic of a "minimum" type of corona. But on the E. side the spreading streamers indicate an intermediate type. The N.E. streamer is remarkable, consisting of a sheaf or bundle of bright filaments, closely packed together as seen in perspective, and traceable for about 26′ of arc from the sun's contour. It is evidently the projection of a great streamer emanating from the solar surface, possibly from the neighbourhood of the large spot that was on the sun's disc. The filamentous structure of the other streamers is also noteworthy. These filaments are superposed upon a bright central ring representing the lower corona. Both the larger and the small-scale photographs agree in showing this decidedly brighter inner coronal ring, sharply defined, and about 1′ to 2′ in height. The slide of the plate-holder for the last larger-scale photograph was only just closed before the emergence of the sun's bright limb, and shows a considerable arc of the chromosphere. Prominences were photographed at the following position-angles:—0°, 11°, 100°, 143°–153°, 235°, 257°–275°, 287°, 353°. They are all small except the fine arched prominence with a bright keystone, between 143°–153°, which rises about 85″ from the limit. Continuous with the prominence of height 50″ at position-angle 353° is a remarkable bright filament in the corona of a different character from the polar rays. In the longest-exposed negative of the larger series it can be traced as far as 7′ from the limb. It is slender, and curves over towards the equator.

With the Zeiss lens Mr. Whitelow secured two plates, at 30 seconds, and 1 minute, after totality was over. These show the whole of the dark moon projected on the luminous background of the corona.

The spectrum of the corona, though extremely faint, is very interesting. It presumably consists of a series of bands or flutings, those in the red being the more prominent.

The provisional wave-lengths of these red bands are:—

1. 6643·9, 6630·5 } three lines.
2. 6530·9, 6502·8 } six lines.
3. 6384·3, 6363·0 } seven lines.



Exposure 25 seconds. 20-foot Coronagraph. Reduced $\frac{1}{10}$.

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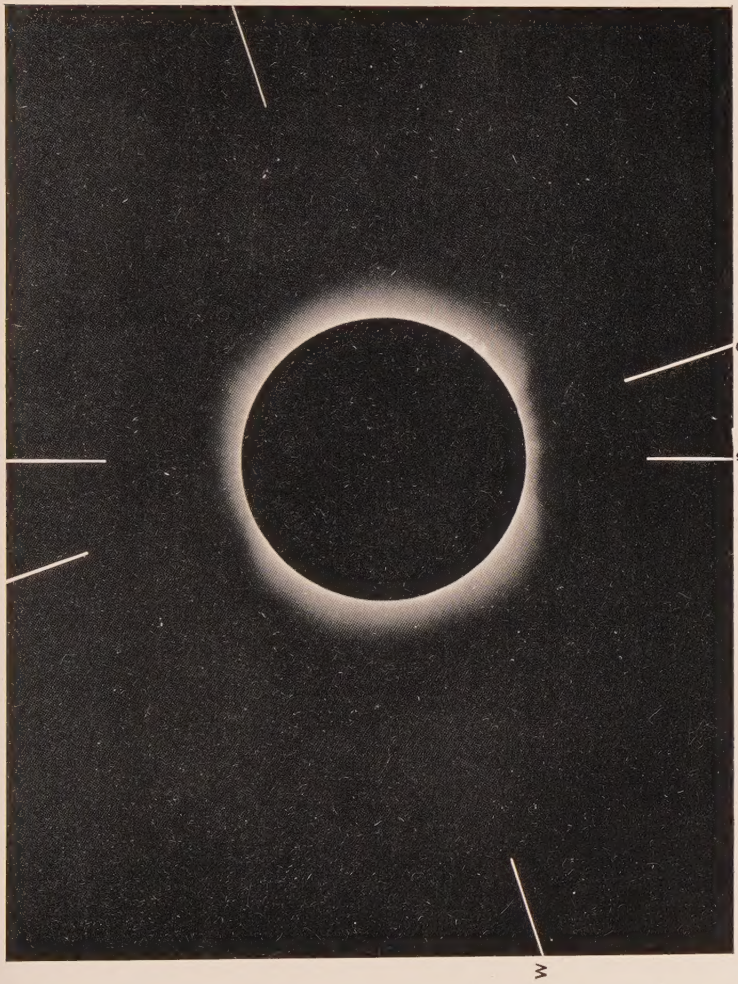


Exposure 7 seconds. 20-foot Coronagraph. Reduced $\frac{7}{10}$.

Outline Diagram of the Main Features of the Solar Corona. 1914 August 21.

The lines, though sharp, are difficult to measure on account of their faintness. The most prominent line in this region is λ 6374·3, occurring in the third band. Other bands are:—

4. 5118·1, 5114·3 } two lines. 5. 5315·3, 5302·7 } four lines.

This last band covers the region of the well-known green coronal radiation. It is present, but extremely faint. There are also probably lines about the following wave-lengths:—5662, 5588, 5544, 5512, 5476, 5227, and 4997, though all these numbers are subject to revision. Seeing that the slit length was $\frac{7}{8}$-inch, and the arc spectrum occupied $\frac{1}{10}$-inch, even if the slit had been placed quite tangentially to the moon's limb about $\frac{1}{10}$-inch would have been covered by the corona on each side of the iron arc. The fact that the coronal bands are separated from the iron arc spectrum indicates that the slit was just inside the image of the moon, and that the outlying portions on each side covered the region of the lower parts of the streamers, but not the lower corona or upper chromosphere. The image of the moon was slightly out of centre on the slit, and just above the central horizontal line.

Stonyhurst College Observatory:
1914 *December* 28.

The Total Eclipse of the Sun, 1914 *August* 21.—*Report on the Kiev Expedition.* By A. Fowler, F.R.S., Major E. H. Hills, C.M.G., F.R.S., and W. E. Curtis, B.Sc.

(*Communicated by the Joint Permanent Eclipse Committee.*)

General Account.

The chief part of the programme of this expedition was based upon observations of the partial eclipse of 1912 April 17, magnitude 0·92, made at South Kensington* and Cambridge.†

These observations showed that the spectrum of the lower chromosphere, or flash spectrum, could be effectively observed at the cusps during a large partial eclipse, or during the partial phases of a total eclipse, for a period ranging up to 30 or 40 minutes. According to the observations at South Kensington, some gain in the visibility of the chromospheric lines became evident when

* A. Fowler, *M.N.*, vol. lxxii. p. 538.
† H.F. Newall, *M.N.*, vol. lxxii. p. 536.

the magnitude of the eclipse had reached about 0·55, while the entire flash spectrum seemed to be visible when the magnitude of the eclipse was 0·8 or greater.

So long as work on the flash spectrum is restricted to a few seconds at the beginning and end of a total eclipse, short exposures can alone be given, and the only instruments of large dispersion which can be employed take the form of slitless spectrographs. While the results obtained with such instruments are of great value, they give no evidence on questions involving a knowledge of exact wave-lengths, since comparison spectra cannot be obtained, and the spectral arcs are of varying thicknesses depending upon the depths of the strata which produce them.

In view of the observations of 1912, however, it seemed possible to employ instruments of greater power than any previously used in eclipse work, and to impress comparison spectra for the accurate determination of wave-lengths. It might thus be hoped to obtain valuable records of the flash spectrum, and data for the investigation of small displacements of the bright lines, due to the solar rotation or other causes, such as were already known in connection with the Fraunhofer lines.

With the aid of the Government Grant Committee, the Joint Permanent Eclipse Committee was able to organise an attempt to secure observations on these lines during the eclipse of 1914 August 21. With the assistance of Dr. Backlund and Professor Donitch (representing the Petrograd Academy of Sciences), arrangements were made for the occupation of a site in the Botanic Gardens of the University at Kiev. Professor Robert Vogel, of the Kiev Observatory, gave much useful information and kindly promised assistance in the orientation and erection of the pillars for the instruments before the arrival of the observers. Further valuable help was promised by Professor Kossonogoff, of the Physical Department of the University, who kindly undertook to supervise the supply of electric current for the comparison spectra, and to assist in the operations during the eclipse, besides providing a number of competent students to take part in the work.

The instruments were dispatched from London on July 18 to the care of H.M. Consul at Riga, Mr. V. H. C. Bosanquet, with the intention that they should be forwarded to Kiev before the arrival of the observers, in order to economise time. The observers themselves left London on July 25, travelling *via* the Kiel Canal and Copenhagen, and reached Riga on the morning of August 1st. Great excitement prevailed in the city in consequence of the European crisis, and was increased later in the day by the declaration of war between Germany and Russia. It was learned from the Consul that it had not been possible to forward the instruments on account of the monopolisation of the railways by the military authorities, and there was no prospect whatever of the party being able to reach Kiev in time to make adequate preparations for the eclipse. Interruption of the telegraphic service made it impossible to seek advice from Dr. Backlund, and it became a question

instruments were determined by Father O'Connor with the theodolite. We were indebted to the energy and engineering skill of Mr. Gibbs for the rapid and efficient mounting of the equipment. No concrete bases were built, the cœlostats being mounted on their packing-cases, loaded with earth and stones, and partially sunk in the ground. Nor were any covering huts for the instruments erected, as experience in heavy rains at Stonyhurst had shown that they could be entirely protected by heavy tarpaulins. These same tarpaulins were utilised at Hernösand, in addition to coverings of waterproof material purchased in the town. The general plan of the positions of the various pieces of apparatus is shown on Plate 9.

The Cœlostats and Coronagraphs.—The 16-inch cœlostat belonging to the Royal Astronomical Society supplied light to three coronagraphs, and to a telescope for projecting an image of the sun on to a screen of ground glass. Much trouble had been experienced with the driving clock of this cœlostat in the expedition to Vavau in 1911. The brake mechanism had been altered by Mr. Gibbs before leaving England, and accordingly it ran much more smoothly. But the driving screw for the sector has a periodic error of one minute, which caused a shift of the solar image projected on the ground glass of the camera about every half-minute, the image then returning to its original position. These errors we thought we had adjusted, as on the day preceding the eclipse, and on the morning of the eclipse day itself, the solar image remained perfectly steady for a period of fully half an hour. But our adjustments of the errors in driving and in the screw were completely upset by the drop in temperature during the eclipse. Luckily the photographs are not seriously marred by this unevenness in driving, but on the longer-exposed photographs there is a slight second image. Mr. Gibbs furnishes the following notes on the cœlostat driving clocks:—

"The driving clock of the 'Hammersley' 16-inch cœlostat has from time to time given considerable trouble. The trouble has been traced to two principal causes:

(*a*) Excessive and variable internal friction.

(*b*) Great slackness in all pivots and joints in the governor.

The former defect arises from weakness in the jointing of the frame or casing of the clock, the securing together of the various parts being not sufficiently definite and rigid. The stresses due to rewinding are occasionally sufficient to disturb the joints and consequently the relative positions of some of the working parts. The latter defect, (*b*), was really the more serious, because the rate of the clock depends on the definite invariable vertical dimension measured from the bottom pivot which supports the weight of the governor to the brake ring or surface: all the joints in the governor and brake levers are much worn and very slack, so that the parts occupy different relative positions from time to time whereby the clock varies its rate so much as to be quite unreliable. The major portion of this difficulty was at Hernösand

obviated by the temporary elimination of those parts of the brake mechanism which were most slack in the joints.

"At Hernösand, after having succeeded in getting the clock to maintain a fairly steady speed, the image of the sun on the ground-glass screen of the 20-feet camera was found to be unsteady, wandering at intervals from the position occupied for the greater portion of the time. This wandering was traced to a defect in the large tangent screw driving the cœlostat polar axis. Observation of the movements of the image disclosed a large temporary disturbance at intervals of one minute, with a smaller temporary disturbance exactly midway between the larger disturbances. It was found that the tangent screw driving the cœlostat made one turn per minute. Careful adjustment to eliminate end-play of the screw failed to cure the trouble, which was finally attributed to some defect of the screw itself, probably a bend. The negatives Nos. 2, 3, and 4 show the effect of these temporary disturbances.

"The driving clock of the 'Grubb' cœlostat also gave some trouble. Inside the barrel of this clock is provided a maintaining gear, but without means for lubricating the internal parts. In consequence, and during the afternoon of August 20, immediately preceding the day of the eclipse, this clock refused to turn at all. Quite a long and tedious examination was necessary before the trouble was traced to the interior of the barrel. The amount of time expended in getting these clocks to work properly was seriously detrimental to the work of the expedition. The advancing or retarding of the cœlostat mirrors by means of slow-motion gear and endless cords or reins is by no means satisfactory, because it requires the use of both hands on the part of the observer. In the case of the 'Hammersley' cœlostat the slow-motion apparatus consisted of a secondary screw and sector operated by means of the endless cord quite independently of the clock-drive, which was therefore not disturbed by the working of the slow-motion gear, and there was no fault to be found in this respect. On the other hand, in the case of the 'Grubb' cœlostat, the slow-motion mechanism, being of the 'mouse-wheel' form, is an integral part of the main driving gear and the 'back-lash' entailed much lost motion, which, moreover, was very unequal in the two directions. Any working of this gear for the purpose of adjusting the 'hour angle' of the mirror disturbed the drive.

"When any adjustment of this mirror amounting to, say, one-fifth of a degree was necessary, it was found better for the observer to go to the clock, throw the mouse-wheels out of gear, and make an approximate adjustment by hand, after which the mouse-wheels had to be replaced in gear before returning to position at the far end of the reins, whence the final adjustment had to be completed.

"It is therefore suggested that for such short runs as are required for eclipse work an electric motor drive would give the most satisfactory results.

"The power required is very small indeed, and small electro-motors are now made to be very reliable and quite reasonably

efficient. A rough calculation of the power may be made as follows:—A weight of 20 lbs. falling 3 ft. in 20 minutes will drive a clock and cœlostat mirror, the rate of power expenditure in this case being 3 ft.-lbs. per minute. It is therefore obvious that a very small motor and battery would suffice. The battery would probably consist of, say, half a dozen small accumulators, and the motor would be geared to the cœlostat polar axis through a train of worm gears. The observer would control the motor from his position at any distance by means of a starter and speed-controller. He would also have by him a speed indicator. The space occupied during transport and the total weight would be considerably less than that of a clock-driving equipment with its weights and tripod for supporting them. The charging of the battery would probably be a matter of no difficulty under any conditions of actual service, and in any case the total running time of the motor (practices included) in the field would probably be well within the capacity of the battery without recharging.

"In case great refinement of speed regulation and checking were deemed necessary, then a speed-checking device could easily be arranged on the following lines:—A dial is provided round which a finger travels. The finger is driven at a known rate by pendulum-controlled clockwork. The dial is driven at the same rate from some convenient part of the cœlostat driving gear when that gear is running at the normal speed. Thus the observer would have a rotating dial and a rotating finger in view, and so long as the motor speed corresponded accurately to the pendulum rate there would be no relative motion of the finger and moving dial, and departure on the part of the motor from the speed as determined by the pendulum would be promptly signalled to the observer by a change in the position of the finger in relation to the dial, which would be provided with marks or divisions for the purpose."

The lens of the 20-foot coronagraph is 4 inches in aperture, and was made by Sir Howard Grubb for the eclipse of 1900, for the Royal Irish Academy. It has been very kindly lent to me by the Council of the Academy—to whom I desire to tender my sincere thanks—for the eclipses of 1905, 1911, and the present eclipse. Exact focussing of the lens was secured by photographs of the fine sun-spot on the sun on the days preceding the eclipse. The sharpest image of the sun corresponded with a diameter of the image of $2\frac{1}{8}$ inches. The camera tube, made under Mr. Gibbs' supervision, was constructed of well-seasoned wooden planks in convenient sections, blackened on the inside, which could be screwed together and held in position by cross battens over the joints. The lengths for the top and bottom of the camera tube were flanged, the sides being screwed into the flanges. It was extremely rigid. A camera bellows with rack-and-pinion motion was fitted at the end of the tube, carrying plate-holders 10 × 8 inches. In order to secure adjustment of the camera in the azimuth of the sun's image on the days preceding the eclipse, the tube was mounted on stout wooden hurdles, so that it could be moved bodily longi-

tudinally. The uprights of these hurdles were sunk in the ground, and additional stability was secured by strong cross battens. The white waterproof covering over the whole instrument acted as a reflector to the sun's rays, and prevented any internal covection currents due to unequal heating. The coronagraph was under the charge of Father O'Connor, assisted by three students—one to hand plate-holders, the other to receive them after exposure of the plates, and the third to make the exposures. A light cord was attached to the wrist of this gentleman, led through staples driven into the side of the camera tube to the observer. He uncovered and covered the lens when the cord was pulled. The following programme for exposures was arranged :—

Number of Slide.	Time by the Eclipse Clock.	Exposure Seconds.
1	128 – 126	2
2	111 – 107	4
3	92 – 82	10
4	67 – 42	25
5	27 – 20	7
6	5 – 2	3

The plates were Ilford Special Rapid, backed, with a speed number (H. & D. scale) 400. All the exposures, except the first, were successfully made.

The two coronagraphs of shorter focal length, namely, the "Abney" camera with a Dallmeyer portrait lens of 4 inches aperture and focal length 34 inches, and the Ross camera, aperture 3·5 inches and focal length 12 inches, were mounted one above the other, *en échelon* with the 20-foot coronagraph. These two cameras were operated by Mr. Whitelow. Four exposures were made with the "Abney" coronagraph, of 10, 50, 15, and 5 seconds respectively, the first two on Ilford Monarch plates, speed number H. & D. 525, and the last two on Ilford Panchromatic plates, speed number H. & D. 400. These four plates were from batches which had very kindly been bathed for me by Mr. Crowther of Carlisle with a special solution to render them impervious to solarisation by long exposures. Hence no over-exposure of the lower parts of the corona relatively to the filmy extensions should be possible. In the Ross camera one long exposure of 95 seconds was made on a Paget "Hydro" plate, which also was unsolarisable by over-exposure. Mr. Whitelow carried out his programme exactly, and in addition made two exposures at 30 seconds and 1 minute after totality, with a Zeiss single "Protar" lens of 14 inches focus working at $f/16$, also on Paget "Hydro" plates. This camera was mounted on a wedge head cut to the latitude of Hernösand, and pointed directly at the sun. These two plates are very interesting, as they show the whole disc of the moon projected on the background of the solar corona.

The Spectrograph and Associated Instruments.—These instru-

ments were under the charge of Father Cortie and Mr. Gibbs, assisted by Herr Askling, the master of mechanical science at the Technical School, Herr Wiberg, the mechanician, who stood by the cœlostat, Herr Helenius, the town electrician, and a student to hand plates. The spectrograph was of the Littrow form, and was put together and adjusted at the Imperial College of Science under the supervision of Professor Fowler. The slit, $\frac{7}{8}$ inch long, belongs to the Royal Astronomical Society; the diagonal prism to the Imperial College of Science. The beam of light, after total reflection at the face of the diagonal, traversed a 6-inch O.G. focal length 98 inches, kindly lent by the Astronomer Royal, and passed through a fine prism of dense flint glass 7 inches edge and refracting angle 40°, made for the Joint Committee by Sir Howard Grubb for the eclipse of 1905. The light after dispersion by the prism was reflected by a 6·5 inches flat mirror, made by the late Dr. Common, and belonging to the Committee. After reflection it passed back through the prism and object-glass, and the spectrum was received on plate-holders placed in a repeating back with rack-and-pinion motion. The plate-holders were kindly lent by Professor Newall. The prism was placed at the angle of minimum deviation and the mirror adjusted so that the red end of the spectrum fell about the middle of the plate, the length of spectrum from λ6700 to λ4800 being 4 inches, and the deviation for H_β being approximately 28·5°. The slit width employed was determined by the method of diffraction bands, which were two in number in the region λ6700, corresponding to a width of 0·0017 inch. The purity of the spectrum was such that the iron line at λ5270, with components differing by λ0·819, was just split. Hence $\frac{\lambda}{d\lambda} = p$ was approximately 6400.

To form an image of the sun on the slit, a Grubb cœlostat, kindly lent by the Royal Irish Academy, with mirror 8 inches diameter, was mounted so that the levelling screws were supported on iron battens fixed to a packing-case firmly embedded in the ground and filled with stones. The cœlostat could therefore be very easily shifted longitudinally out of the meridian. This was necessary, as the heavy spectrograph mounted in its packing-case, with removable panels over the optical parts, was placed once for all in the azimuth of the sun's image at the time of totality. Therefore, in the days preceding the eclipse, we had to shift the cœlostat bodily out of the meridian towards the east, to obtain a horizontal beam of sunlight on the slit. The cœlostat slid along the iron supporting battens, until on the day of the eclipse it lay with its axis in the meridian. An Alvan Clarke lens of 5 inches aperture and 7 feet focal length, belonging to the Stonyhurst Observatory, gave an image of the sun on the slit plate $\frac{13}{16}$ of an inch in diameter. The lens was placed just in front of the mirror in the prolongation of the azimuth of the sun's image at totality. It was adjustable in azimuth.

The Comparison Spectrum.—A novel feature in eclipse opera-

tions was the placing of a comparison spectrum of the iron arc on the plates taken before and after totality, and on the plate of the coronal spectrum taken during totality. The electric current was carried from the mains at the Technical School by two long leads laid along the ground and connected to an iron-arc lamp brought from Stonyhurst, a variable resistance being inserted in the circuit. The pressure was 110 volts and the current 12 amperes. We are much indebted to the town electrician, Herr Helenius, for lending the leads and the resistance, and for making all the necessary connections free of charge. A single lens 4·9 inches aperture and 11·4 inches focal length formed the image of the arc on the slit, the beam of light being bent through a right angle by a diagonal prism. This prism moved in guides in front of the lens, so as to be readily moved forwards when it was desired to place the image of the arc on the slit. The distances were so adjusted that the ratio of the aperture of the lens to the distance of the slit from the lens should equal that of the aperture of the O.G. of the spectrograph to its focal length, or distance from the slit. By this means the solid angles subtended by the two lenses on each side of the slit were equal, and the O.G. of the spectrograph was filled with light from the arc. The following device was adopted to place the iron-arc spectrum on the slit during totality. The image of the eclipsed sun was placed by Mr. Gibbs on the slit-plate, so that the slit was almost tangential to the dark image of the moon on the east or advancing side, and intersected the corona on each side of the moon's image. The first exposure on the corona lasted for about 60 seconds. The iron arc having then been switched on by Herr Helenius, the diagonal prism was pushed forward into position by Mr. Gibbs, and the slit was covered by a plate of zinc, which hung ready, suspended by a wire from a screw in the packing-case of the spectrograph. This shield of zinc had in it a horizontal slot $\frac{1}{10}$-inch wide, which uncovered the dark image of the moon centrally to that extent, and permitted the light from the iron arc to pass through the same length of the spectrograph slit. The light of the iron arc was thus superposed on the dark image of the moon. The iron arc was exposed for 4 seconds, and the diagonal being pushed back, and the zinc slotted plate removed, the exposure on the corona was continued for another 40 seconds, neither plate, nor cœlostat mirror, nor image-forming lens having been disturbed in the meanwhile. The operations of adjusting the diagonal prism, covering the slit with the slotted zinc plate, and striking the arc took about 20 seconds.

In addition to this photograph six exposures on the tip of the southern cusp on the sun's east limb were commenced 7 minutes before totality, a seventh and middle exposure being made on the iron-arc. A precisely similar set of exposures were made 5 minutes after totality, but this time on the tip of the south-west cusp. The exposures were about 1 second each, at intervals of about a second, the slit being covered and uncovered by a shield of zinc

held in the hand by Herr Askling. Seven successive exposures were likewise attempted at the first "flash." These had been timed to commence 10 seconds before totality, and to continue 4 seconds after totality. They actually started 12 seconds before totality, owing to an error in the signal, and were finished about 3 seconds before totality, the successive exposures on the plate by means of the repeating back being made too fast. Consequently the spectra shows only bright C, D_3, and F, and the change from the dark line to a bright line spectrum. This plate may probably be regarded as a failure. All the spectra were photographed on Wratten & Wainwright's Panchromatic B plates, a freshly bathed set of plates having been sent out to me to Hernösand by that firm.

The Eclipse Clock.—This consisted of a set of spring-driven clockwork with a fan governor, which permitted of a sufficiently accurate adjustment of the rate, so that one revolution of the projecting arbor was made in the estimated number of seconds of totality. The arbor carried a hand about 12 inches long, and a dial with a divided circle two feet in diameter was arranged in appropriate position so that all observers could conveniently see it. The dial was illuminated during the eclipse by means of an incandescent electric lamp.

The circle on the dial was not divided into equal spaces, neither were the divisions numbered; the system employed was a series of dark segments of various lengths alternating with blank segments. The passage of the hand over each of the dark segments indicated the commencement, duration, and end of each exposure made with the 20-feet coronagraph. The passage of the hand over each of the blank spaces indicated the time reserved for slide-changing and the preparations for the succeeding exposure.

The clock was started the moment totality commenced, that moment being determined by Father O'Connor, who watched the projected image of the sun on a transparent screen arranged behind the eyepiece of the 3-inch Cooke telescope. This telescope and screen were supported on the tube of the 20-feet coronagraph and were fed with light from the 16-inch cœlostat. At the moment Father O'Connor saw that totality had commenced he called out "Go," and simultaneously pulled the cord which released the clockwork. The hand then started to make a single revolution round the dial during the estimated time of totality.

3. *The Day of the Eclipse and After.*

During our three weeks' stay at Hernösand, every day had been cloudy except August 9, and the day of the eclipse. Most days were quite clear in the early part of the morning, and then clouds would form, and cover the sun at the time at which eclipse would take place. There were clear intervals in the afternoon, and the clouds would disappear at night. All this time the wind was in the N.W. or N.N.W. quarter, and we experienced a certain

number of showers of rain, though there was no day of continuous rain. The barometric height was very constant, about 29·5 inches, and all these types of weather conditions accompanied the same direction of the wind and the same height of barometer. But on August 9 a beautifully clear sky had accompanied a S.E. wind. On the evening of August 20 the wind again changed to that quarter, with a slight rise in the barometer. A beautiful night heralded the absolutely perfect weather conditions for the day of the eclipse, although clouds formed in the afternoon, about an hour after the eclipse was over. In general the atmosphere at Hernösand was very translucent.

Several drills had been held each morning, commencing on August 18, the observers being drilled separately at each instrument and then all together. Seven students of the Technical School under the direction of Herr Askling assisted us, in addition to Herr Wiberg and Herr Helenius, whose duties have already been mentioned. Three of these students, with Herr Askling, made a composite drawing of the corona, on specially prepared cards, on which concentric circles had been drawn round a dark disc representing the moon, at successive radial distances apart of $\frac{1}{8}$ a lunar diameter for the first diameter and $\frac{1}{4}$ for the second. During the drills they made sketches from a drawing of the corona of 1901, each taking one quadrant only.

The duration of totality had been calculated from the formula, page 2, *Nautical Almanac Circular* No. 20, based on the path of the moon's shadow as published by Dr. Crommelin in the *Journal of the British Astronomical Association* for January 1914. This path was plotted on the Swedish General Staff Maps for Hernösand, and gave the distance of our station at the Technical School from the central line of total eclipse as 3·9 miles. The duration of totality under these circumstances was reckoned to be 129·5 seconds. It was actually about 4 seconds less.

A goodly number of spectators assembled in the field about one hour before totality, and watched the progress of the eclipse through darkened glasses. To prevent any inadvertent intrusion we roped off a large enclosure round the instruments. The spectators were extremely quiet, and no trouble was experienced from their presence. At 10 minutes before totality, as observed on the projected image of the partially eclipsed sun on the ground-glass screen by the method of cusps, Father O'Connor blew three blasts on a whistle. At the signal the observers and their assistants stood to their stations, and the cœlostat clocks were wound up. At 5 minutes before totality, at the signal of two blasts on the whistle, the lamps were lit and the slides drawn. At 10 seconds nominally, but in reality at 12 seconds before totality, one blast was blown, the signal for exposures on the "flash" spectrum. At the signal "Go," the string was pulled which released the mechanism of the eclipse clock, and the exposures with the coronagraphs were started.

The naked-eye view of the corona in a perfectly transparent

whether an attempt should be made to carry out the programme of observations at Riga or to go to Sweden and join the party at Hernösand.

The difficulties in the way of work at Riga seemed to be insuperable. There was apparently no one at liberty to aid the observers in finding a suitable site, or to give the assistance which would be necessary in setting up the instruments and carrying out the rather complex programme of operations during the eclipse. A further serious difficulty was caused by the fact that most of the funds of the expedition had been made payable at Kiev, and could not be obtained at Riga. Under these circumstances it was decided to go to Sweden if there should be an early opportunity of doing so, and to rely upon being able to share the facilities afforded to the observers at Hernösand. Information was received that a steamer would leave for Stockholm as soon as permission could be obtained, and the observers went on board; but the expected permit did not arrive, and it seemed that they must of necessity remain at Riga if the eclipse was to be observed at all.

The situation became more acute when our own country became involved in the war. On the morning of August 6, through the British Embassy in Petrograd, Major Hills was recalled home for military service, and secured a passage in a cargo steamer which was to leave on the same day for Copenhagen. Without his assistance it was evident that the chief purpose of the expedition could not be achieved, and it did not seem worth while for the two remaining observers to take the risk of indefinite detention in Russia in order to carry out the minor parts of the programme. This view was emphasised by the fact that the weather at eclipse time had been very unfavourable on four out of five days. The expedition was therefore abandoned, and the whole party proceeded to Copenhagen, where they arrived on the evening of August 9. There was some delay at Copenhagen, but Major Hills reached England *via* Leith on August 14, and Messrs. Fowler and Curtis, *via* Bergen and Newcastle, two days later.

As the proposed observations were to have been attempted for the first time, it may be of interest to give a brief account of the instruments which it was intended to employ, with some details as to the operations during the eclipse.

Instrumental Equipment.

The dispersive part of the apparatus was the 10-feet concave grating spectrograph of the Imperial College laboratory, of which a full account has already been published.* The mounting is on the Littrow principle, which, for the purpose in view, has the advantage of giving comparatively little astigmatism, besides the great convenience of being wholly enclosed. The grating was adjusted so as to give the third order spectrum from $\lambda 3640$ to

* A. Eagle, *Astrophys. Jour.*

λ4400 on a plate 18 inches × 3 inches, giving a mean dispersion of 1·69 A.U. per millimetre. The definition in the region selected was extremely good, and the brightness was such that exposures of from 5 to 10 secs. were ample for the iron arc with the Lumière "Violet Label" plates employed.

For the purpose of the eclipse observations, mirrors were introduced inside the instrument so as to reflect the green region of the first order spectrum into an eyepiece conveniently placed near the slit. The exposing shutter, in front of the photographic plate, was arranged so that the iron comparison could be placed between two solar spectra, the latter being produced simultaneously on account of the astigmatism of the grating.

The image of the sun on the slit was formed by a Zeiss triple object-glass of 5 inches aperture and 80 inches focal length, in combination with a negative lens which enlarged the image to about 2 inches diameter. Further enlargement would doubtless have been an advantage, but the condition that the grating must be filled with light required that any increase in the size of the image should be accompanied by an increase in the diameter of the object-glass, and no other conveniently mounted lens was available. In order to bring any required point of the sun's limb tangentially on the slit, an arrangement for rotating the image was provided. This consisted of three small plane mirrors, placed near the magnifying lens, on the side towards the object-glass, one parallel to the axis of the telescope and the other two equally inclined and a short distance away from it. The ordinary telescope tube was used as a support for the main object-glass, but the eyepiece end was reconstructed so as to permit the easy rotation of the rotator and magnifier together.

Sunlight was brought into the telescope by a 12-inch cœlostat, and a secondary mirror for correcting the changing direction of the horizontal beam from day to day. The fine adjustment of the cœlostat mirror, and a screw for vertical adjustment of the second mirror, were under the control of an observer near the slit through the medium of long rods attached to them. The guiding was to be controlled by direct observation of the image on the slit, using a special low-power microscope of long focus, and by occasional observations of the first order spectrum.

The comparison spectrum was obtained from an iron arc, a magnified image of which was thrown on the slit by an ordinary condenser of about 7 inches focal length, after reflection by a right-angle prism which could be moved in or out of position, as required, along guides in a direction at right angles to the axis of the optical system producing the sun's image.

For convenience of transit and use, the packing-case containing the spectrograph, measuring 13 ft. × 26 ins. × 24 ins., was so constructed that it was unnecessary to remove it until the instrument was returned to South Kensington. It was made of stout material, and was provided with convenient openings which gave the necessary access to the grating, camera, and slit.

The entire apparatus, with the huts for the spectrograph and cœlostat, was erected and tested in the open ground before dispatch to Russia, and was considered very promising.

As the large spectrograph could not be expected to be of use during totality, a quartz spectrograph of moderate dispersion was arranged for work on the spectrum of the corona. It was especially desired to record as much as possible of the ultra-violet spectrum with this instrument, and it was accordingly fed by a cœlostat having a mirror of speculum metal, which was 4 inches in diameter. The image of the sun was formed by a 2-inch concave mirror of 24 inches focal length, also of speculum metal, kindly lent by Mr. W. Shackleton.

A portable equatorial telescope of 3 inches aperture, and a small spectroscope for the observation of prominences, also formed part of the equipment. It was thought that this combination might be of use in the work preparatory to the eclipse by giving indications of the locality of bright prominences, on which trial exposures with the grating spectrograph would have been instructive. A knowledge of the positions of prominences on the day of eclipse would also have been necessary in connection with the discussion of the spectroscopic results.

Among minor pieces of apparatus was a large direct-vision spectroscope, with the collimator removed, with which it was intended to observe the coronal rings during totality.

Choice of a Station.

Observations of the kind contemplated with the grating spectrograph call for careful consideration of several questions, among them, independently of accessibility and probable weather conditions, that of the most suitable station for observation of the eclipse. In his report on the observations of 1912, Professor Newall pointed out that there might be an advantage in choosing a place just outside the zone of totality, where the magnitude of the eclipse would be, say, 0·99. Under these circumstances, the change in the position-angle of one of the cusps would be slower during a considerable part of the useful phases than at a place on the central line, and the difficulty of guiding on the cusp would consequently be lessened. The fact that observations of the corona with other instruments could not be undertaken by an expedition so situated, however, would be a serious disadvantage.

The selection of Kiev, which was nearly on the central line, was determined primarily by the general facilities offered by the University, and especially by the provision of continuous electric current for the iron comparisons. Nevertheless, it was believed that with the guiding arrangements which had been prepared it would not have been difficult to keep the cusp on the slit during the greater part of the eclipse. The undesirability of covering any great range of latitude in a single exposure was of course recognised, but there was little danger of this except near the

beginning and end of totality, and it was proposed to modify the procedure during these periods.

The data relating to the cusps, as calculated for Kiev, are given in the following table. The position-angles are expressed as heliographic latitudes, and the intervals are counted from the beginning of totality. The last column shows the projection at the cusp of a stratum 1″ in depth, as measured along the moon's limb. For corresponding intervals after totality, the results are the same, except that the third and fourth columns are interchanged and the signs and azimuths reversed.

Interval before Totality.	Mag.	Lat. of Cusps.		Projection of 1″-Stratum.
		Upper.	Lower.	
30 mins.	0·57	+83 W	−49 W	1·26
20 ,,	0·71	+87 E	−59 W	1·74
15 ,,	0·78	+82 E	−64 W	2·24
10 ,,	0·86	+76 E	−70 W	3·21
5 ,,	0·93	+67 E	−79 W	5·84
1 ,,	0·986	+43 E	−77 E	19·5
30 secs.	0·993	+31 E	−65 E	30·0
15 ,,	0·997	+19 E	−54 E	44·0
0 ,,	1·000	−17 E	−18 E	

Similar data were calculated for stations just on the edge, and just outside, the zone of totality. For the upper cusp, at intervals before central eclipse corresponding to those of the table, the results, which may be of interest for comparison, were as follows:—

	30 m.	20 m.	15 m.	10 m.	5 m.	1 m.	30 s.	15 s.	0 s.
Edge of zone	+81 W	+90	+85 E	+81 E	+77 E	+74 E	+73 E	+73 E	+73 E
Outside ,, (mag. 0·99)	+82 W	+89 W	+87 E	+84 E	+84 E	+78 W	+69 W	+63 W	+55 W

As the most valuable time for work on the flash spectrum would be during 15 minutes before and after central phase, when the magnitude of the eclipse was 0·8 or greater, these figures suggest that the advantage would, on the whole, have been with a station on the edge of the belt of totality. Kiev, however, was chosen for reasons already stated.

There is another consideration which might be held to justify the selection of a station near the central line. If observations are to be made only on that part of the chromosphere which projects at the cusp, it is evident that a greater length of slit would be illuminated if the slit were tangential to the moon than if tangential to the sun. A rotation of the image about the centre of the moon, and not of the sun, was thus suggested. Calculations showed that this would also result in a reduced rate of rotation required to keep the image of the cusp on the slit. For the period before totality the data for Kiev were as follows, the

position-angles being reckoned on the moon's limb in the ordinary way from the north point through east:—

	30 m.	20 m.	15 m.	10 m.	5 m.	4 m.	3 m.	2 m.	1 m.	0 m.
Upper cusp	63°	55°	50°	50°	47°	51°	54°	58°	68°	125°
Lower cusp	187	195	200	200	203	199	196	192	182	125

On comparing these figures with the corresponding figures in the first table, it will be seen that during the most critical interval, from 15 mins. to 2 or 3 mins. before totality, there would have been a substantial reduction in the rate of motion to be given to the image rotator. It would probably have sufficed in fact to set the image for position-angle 50° at 20 mins. before totality and to leave it undisturbed until near totality, the necessary remaining adjustment of the image being made by means of the slow-motion rods of the cœlostat and secondary mirror. Hence it was proposed to adjust the polar axis of the cœlostat to the pole of the moon's apparent diurnal path at the time of eclipse, and to regulate the driving clock to lunar rate. The rotator would then turn the image about the moon's centre.

It must be admitted there is still some uncertainty as to the most effective position for the slit with respect to the sun's image for the observations proposed. If the visibility of the bright lines depended solely upon the reduced sky illumination, a slit tangential to the sun would be equally effective at any point on the sun's limb. But the observations at South Kensington in 1912 indicated that this was not the case, and that observations on the chromosphere projecting at the cusp were most effective, possibly on account of the reduced tendency for atmospheric tremors to cause an excess of photospheric light to enter the slit. A slit tangential to the moon, and only illuminated by that part of the chromosphere projecting beyond the cusp would seem to be the ideal position, and is only open to the objection that the length of slit illuminated would be small. It is possible that a slit tangential to the sun at the cusp, illuminated in part by the chromosphere adjacent to the sun's limb, would be better in practice. An attempt was made to investigate this point during the eclipse of 1914 August 21 at South Kensington, but the sky was too hazy and the magnitude of the eclipse (0·65) too small to permit satisfactory conclusions to be drawn.

Proposed Procedure During Eclipse.

From the data previously given it will be observed that the upper cusp appeared in high northern latitudes during the greater part of the time available for useful observation before totality, while the lower cusp similarly occupied high southern latitudes after totality. In view of the desirability of obtaining some spectra at points in lower latitudes, and in consideration of the rapid motion of the cusp near totality, it was proposed to vary the procedure near the beginning and end of totality and to place the

image so that latitude 0° E. fell tangentially on the slit before, and latitude 0° W. after totality. It was thus hoped to obtain spectra near the sun's north and south poles, and at opposite points of the equator, in each case with iron-arc comparisons. The slit was provided with a shutter for regulating its length at different phases, as it was evidently undesirable to expose a length much greater than that of the chromospheric arc to be photographed.

One of the greatest difficulties anticipated was that of determining the exposures necessary for the bright lines. In the absence of any previous experience, it was not possible to draw up a definite programme in advance, and it was therefore intended to estimate the exposures by observation of the brightness of the first order spectrum with the arrangement provided for this purpose, as compared with the iron arc for which the exposures were known. A shutter was provided so that a plate in readiness for exposure could be rapidly covered or uncovered during this operation. In order to avoid total failure through misjudged exposures, some of the plates were to have been developed as the eclipse progressed, and a dark room for this purpose, and for the changing of plates, was attached to the hut containing the spectrograph.

Five exposures could be made on each plate if desired, but only one would usually have been made, especially on the earlier plates, unless a previous exposure had been considered unsatisfactory through errors in guiding. The general plan was thus to make a series of exposures on the sun and iron arc, alternately, on the upper cusp until 2 or 3 minutes before totality, when the image was to be turned so as to bring the eastern point of the sun's equator on the slit. During totality, an iron-arc spectrum was to be exposed in connection with the previous exposure on the sun, the rotator turned so as to bring the western point of the equator on the slit, and another iron arc exposed in readiness for the next exposure on the sun. It will be clear that the various operations would have kept at least three observers fully occupied.

In addition to those already named in the course of this report, thanks are due to the authorities of the Imperial College for the loan of instruments; and to Mr. J. W. Colebrook, Superintendent of the Physics Workshop, for invaluable aid in the adaptation of the various instruments for the special purposes of the eclipse observations. Also to Messrs. Lumière for supplying special batches of "Violet Label" plates for use during the eclipse. Mr. V. H. C. Bosanquet, H.M. Consul at Riga, rendered valuable aid by taking charge of the instruments on their arrival at Riga, where they still remain under his care.

Total Eclipse of the Sun, 1914 *August* 21. *Preliminary account of the observations made at Minsk, Russia.* By H. S. Jones, B.A., B.Sc., and C. R. Davidson.

(*Communicated by the Astronomer Royal.*)

I. General Arrangements.

The Lords of the Admiralty having sanctioned the dispatch of an expedition from the Royal Observatory to observe the total solar eclipse of August 21, it was arranged, in conjunction with the Joint Permanent Eclipse Committee, that the party should proceed to Minsk in Russia, a large town lying very close to the predicted line of totality, and about midway between Riga and Kiev.

The observers from Greenwich who took part in the expedition were Mr. H. S. Jones and Mr. C. R. Davidson. Mr. P. H. Hepburn generously volunteered his assistance, and shared in all the work connected with the erection and adjustment of the instruments, as well as in the observations on the day of the eclipse. This help was particularly valuable, inasmuch as, owing to the outbreak of war, we were unfortunately deprived of other assistance which had been promised us.

We were afforded all facilities by the Russian Government for the rapid conveyance of our instruments from Petrograd to Minsk and back, a privilege of especial value at a time when, owing to the war, no freight of any kind was ordinarily being accepted. We are greatly indebted to Dr. Backlund, Director of the Observatoire Central Nicolas, Pulkovo, for his very generous help freely given in many ways, both before and after our arrival in Minsk, to M. A. Amaftounsky, Chief of Control for the Government of Minsk, for making all arrangements for a suitable observing station near Minsk, and for much help given during our stay there, both in an official as well as an unofficial capacity; to Dr. and Mme. Kodis and their sister-in-law Mme. Krzyzanowska for the use of their house and garden as an observing station, for much help and thoughtful provision for the comfort of the observers, and for assistance on the day of the eclipse; and also to MM. Borodin, Nowicki, and Szymanowski for assistance in carrying out our programme of observations.

Itinerary.—We left London on July 17, accompanied by Mr. Hepburn, travelling direct to Petrograd by the St. Petersburg Express Line steamship *Imperator Nikolai II.*, the instruments and the frameworks of the observing huts being conveyed with us. Petrograd was reached on July 22, and as soon as the necessary customs formalities were completed we proceeded to Minsk, arriving there on July 30. There was some fear that the outbreak of war would delay the arrival of the baggage, but it fortunately came a few days later. After the eclipse we left Minsk for Petrograd on August 26, and Petrograd for England on September 1, the return journey being made *via* Finland, Sweden, and Norway to Newcastle-on-Tyne, where we arrived on September 7.

Mr. Hepburn preceded us by a few days. The instruments were left at the Observatory, Pulkovo, in charge of Dr. Backlund, to be forwarded after the end of the war.

Station.—The station occupied was near Minsk, a town on the main railway line from Warsaw to Moscow, and roughly midway between them. The site for the erection of the huts and instruments, which had been chosen for us by M. Amaftounsky, was the garden of the country villa of Dr. Kodis, the chief surgeon in the town hospital in Minsk. The situation was excellent, being on the summit of a slight eminence in a very flat tract of country, so that an uninterrupted view was obtained in practically every direction. The site was about 3 miles north from Minsk, and the predicted central line passed about $1\frac{1}{4}$ miles to the N.E. Its position, taken from an ordnance map, was 53° 56′ 20″ N. lat. 27° 31′ 57″ E. of Greenwich.

Erection of Instruments, etc.—The instruments were brought from the railway station to the camp, a distance of $4\frac{1}{2}$ miles, in small waggons on August 2. The following day the erection of the huts to cover the instruments was commenced. The huts consisted of light wooden frameworks, each piece being marked so that they could be erected quickly, and these were covered with Willesden waterproof canvas. One hut covered the Thompson coronagraph and the Simms and Franklin-Adams photographic cameras, a second covered the Hills spectroscope, whilst the third served as a developing room. The first two were constructed in two parts, moving on runners, which were separated when in use sufficiently for the cœlostat mirrors to be fully illuminated by the sun. When not in use they were bolted together. To avoid the delay and trouble incidental to the construction of brick or concrete piers for the instruments, rigid wooden tripods were constructed at Greenwich for carrying the cœlostats; each piece was marked and they were fitted together on the ground. Each cœlostat was mounted upon such a tripod, which was sunk a few inches into the ground, whilst the driving clock was mounted alongside on a separate stand. The clock weights of the two 16-in. cœlostats were carried over pulleys fixed to the cross-bar of the posterior half of the hut, and the independent mounting of the driving clock ensured that any slight motion which it might receive, due to the shaking of the hut in wind, was not transmitted to the cœlostat. These tripods were found to be perfectly satisfactory, the cœlostats being quite rigid. Other wooden tripods and frameworks, which were also fitted together on the ground, were constructed for carrying the other instruments. These frameworks were not elaborately finished, the idea being that when they had served their purpose they should be abandoned, and the trouble of repacking them saved. The sun at eclipse time was very conveniently situated for observation. The angle of inclination upon the cœlostat mirrors, in order that the reflected light should be horizontal, was only 22°, and the points of second and third contact on the reflected sun were on a diameter inclined to the

horizontal at an angle of less than 1°; it was, therefore, not necessary to give the spectroscope any tilt, and the coronagraph and the colour-screen cameras were only given a slight tilt sufficient to make the plate-holders come at a height convenient for the observer.

Meteorological Conditions.—During the twenty-one days spent in Minsk previous to the eclipse, the weather was of a very uniform type. With only one or two exceptions, the day commenced with the sun shining and the sky clear. During the course of the morning cumulus clouds gradually collected, and the maximum cloudiness was attained in the early afternoon (*i.e.* about eclipse time), when seven-eighths or more of the sky would be clouded. Towards evening the clouds dispersed again, and the nights were generally remarkably clear. On only two days preceding the day of the eclipse would it have been possible to have made any observations.

Personnel.—The following list gives the names of those who took part in the observations:—

H. S. Jones—Thompson coronagraph. Large scale photographs of corona.

C. R. Davidson—Hills quartz spectroscope. Spectrum of "flash" and of corona.

P. H. Hepburn—Assisted Mr. Davidson, making the exposures and changing plate-holders, etc.

Leon Szymanowski—Astrographic Triplet 6″ prismatic camera. Photographs of corona through green colour screen.

Alexander Borodin—Simms 6″ prismatic camera. Photographs of corona through green colour screen.

Leon Nowicki—Read thermometer during eclipse.

Mme. Kodis—Counted seconds during totality.

Mlle. Kodis—Watched for shadow-bands on a white sheet.

The mode of procedure, which was carefully rehearsed on several occasions previously, was as follows:—The observers were stationed at their instruments, and Mr. Jones watched the diminishing crescent of the sun on the ground glass of the coronagraph. He had a paper on which the lengths of the crescent were marked, computed for the intervals 3 mins., 2 mins., 1 min., 30 secs., 20 secs., and 15 secs. before totality. At 3 min. before totality a signal was given to Messrs. Davidson and Hepburn to cease work on the attempt to obtain the "flash" spectrum by exposing on the cusps, and to make ready for the flash proper. Signals were also given at 2 mins., 1 min., 30 secs., and 15 secs. before totality. At 15 secs. before totality "Get ready" was called, which was the signal to the observers with the spectroscope to commence the exposures for "the flash" and for the other observers to be ready: the first plate was then inserted in the coronagraph. The signal "Go" was given by Mr. Jones, watching the rapidly diminishing crescent, immediately the last bright bead was seen to disappear. Upon this signal being given Mme. Kodis started a metronome, rated to give seconds, and counted in French each second throughout

totality. During totality, exposures at the several instruments were made, as described below.

The Day of the Eclipse.—It was quite clear in the early morning, but as on other days cumulus clouds began to collect later, and the experience of those days did not tend to make the expectations of the observers run high. There was a strong gusty wind from the N.N.E.

However, there were occasional clear patches of sky, and during one of these clear intervals the first contact was seen, and the chronometers adjusted by it. It had been arranged with Dr. Backlund that Dr. Bonsdorff from Pulkovo should join us in Minsk with a small meridian circle, in order to make time determinations and to observe the times of the four contacts during the eclipse, but the outbreak of war had rendered this plan impracticable. During the last half-hour preceding totality, the sky was somewhat clearer. When totality commenced there was a great deal of cloud in the sky, but the eclipse was seen through a clear break which lasted until the end of totality, immediately after which the returning crescent was veiled by thin cloud. M. Amaftounsky in Minsk had been intending to draw the corona with the aid of his telescope for comparison with the coronagraph photographs, but he unfortunately obtained only a fleeting glimpse of it. The atmosphere during totality seemed very transparent, and although shadow bands were looked for, none were seen. A noticeable feature was the sudden drop in the wind following upon a very strong gust at the commencement of totality, and the almost complete stillness during totality.

Dr. Kodis had intended to observe the duration of totality with a stop watch, but being also engaged in watching for the appearance of a bright-line spectrum with a small direct-vision spectroscope, he forgot to start the watch, and no record was obtained.

The following readings of dry- and wet-bulb thermometers were taken by M. Nowicki on the eclipse day :—

	Pulkovo Time.	Dry-bulb °F.	Wet-bulb °F.
	h m		
	12 0	56·1	50·5
	13 0	58·1	52·4
Beginning of eclipse	13 25	58·9	53·3
	13 45	60·4	54·6
	14 0	57·4	51·5
	14 15	57·5	51·7
	14 30	56·5	50·3
	14 35	56·0	50·0
Totality 14 35 48 – 14 38 1	14 37	55·7	49·8
	14 45	55·0	49·0
	15 0	55·2	49·6
	15 15	56·3	50·5
End of eclipse	15 30	59·0	53·0

The minimum temperature occurred some minutes after totality, and was 55°·0. The fall of temperature was not large, and the humidity does not appear to have been much affected.

The darkness during totality was not very intense. Venus and Mercury were seen, both shining very brightly. Regulus was also seen through the corona. The corona appeared of a steely-blue colour. The polar plumes were distinctly seen, but no prominences.

Development of the Photographs.—As stated above, a fully equipped dark-room was taken to Russia. The intention had been to develop the small colour-screen photographs and the spectroscope films as soon as possible after the eclipse, but to bring the large coronagraph photographs back undeveloped to Greenwich, where they could be more conveniently manipulated. However, it was thought wiser not to attempt to carry undeveloped photographs about during war time, and all the photographs were accordingly developed before leaving Minsk. The developing hut used to get very hot during daytime owning to the lack of ventilation, so that all the development was done at night-time, on the night of the eclipse day and succeeding nights.

II. Photographs of the Corona.

The programme of observation was composed of two distinct parts:—

(1) Photographs of the corona on a large scale to show structural detail.

(2) Photographs on a small scale through green-coloured filters, to obtain evidence as to the distribution of coronium in the corona.

(1) *Large-scale photographs.*

The instrument used was the Thompson photographic telescope, with object-glass of 9 inches aperture, and focal length 8 feet 6 inches, belonging to the Royal Observatory, in combination with a Dallmeyer concave telephoto lens of 4 inches aperture, and 16 inches focal length, fitted as a secondary magnifier, to give an image of the sun 4 inches in diameter, with a field (for full pencils) of 14 inches. The total length of the coronagraph was 12 feet, the equivalent focal length being 36 feet. The coronagraph was focussed in the same manner as for previous eclipses * by means of the image of an object (gauze net) in the plane of the plate reflected from the plane mirror of the cœlostat. The cœlostat used, which was loaned by the Joint Permanent Eclipse Committee, had a 16-inch plane mirror, made by Dr. Common. The coronagraph pointed to the mirror at an angle of depression of about 3° or 4°, and was at an azimuth of about 69° W. of S.

* *Monthly Notices, R.A.S.*, lvii. p. 105; *Proc. Roy. Soc.*, lxiv. p. 8.

for the day of the eclipse. The camera was furnished with eight plate-holders to take 15 × 15 inch plates, or for the shorter exposures 12 × 10 inch plates in an adapter.

The seven plates for photographs of the corona during totality were exposed as below, the times counting from the commencement of totality:—

No.	Exposure. Beginning.	End.	Duration.	Plate.	Developer.
	s	s	s		
1	2	4	2	Process 12 × 10	Pyro-soda.
2	14	22	8	Empress 15 × 15	Hydroquinone (dilute).
3	32	42	10	Special Rapid 15 × 15	,,
4	52	67	15	,, ,,	Pyro-soda.
5	77	102	25	,, ,,	Rodinal.
6	112	117	5	Empress 15 × 15	Metholquinol.
7	127	129	2	Process 12 × 10	Pyro-soda.

An eighth plate was taken shortly after totality and exposed twice on the sun for orientation (with the driving clock stopped for 1½ min. between), the exposure being as short as possible, and the aperture reduced to 1 inch. The plates were made by Messrs. Ilford, Ltd., the Special Rapid plates being treated by a special process to avoid reversal by over-exposure. This treatment is not recommended for use in future eclipses, these plates, one in particular, being disfigured with marks apparently due to the bathing solution not having been caused to flow evenly over the plate during treatment: the disfigurement fortunately does not materially interfere with the coronal detail.

The exposures were made by means of an exposing cap, attached to the end of the coronagraph, and operated by a cord from the other end. The slight jarring of the coronagraph due to exposing in this way is perceptible upon the short-exposure plates, and it would have been better to have had an observer to give the exposures at the object end of the coronagraph. However, owing to the war, we were short of help, and we were unable to arrange this. On some of the photographs, also, the definition of the east and west limbs of the sun is not all that might be desired, owing to a slight motion of the image in right ascension, caused by slight irregularities in the driving of the cœlostat. The reflected image was found to have a period of one minute, composed of a slow creeping forward of the image for about 40 secs., and then a quicker run back in about 20 secs. This period of one minute is the period of revolution of the driving screw, and was traced to the driving arc riding up in one particular position of the screw. It was found impossible to remedy this defect completely. Apart from these slight imperfections, the photographs are excellent. The programme of the exposures, and the method of development, has resulted in a well-graduated series of photographs. No. 1 shows a small

amount of corona, and several prominences on the east limb of the sun, including a very large, fine-arched prominence. No. 2 has been so developed as to give a thin negative, showing well the inner corona, the polar plumes, and the prominences. No. 6, with an equal exposure on a similar plate, was developed with a vigorous developer so as to show the form of the corona and its structure. Nos. 3, 4, 5 show the corona in extension, and also many interesting details in the structure; of particular interest is a coronal arch proceeding from a large prominence in the S.E. quadrant. No. 7 has just caught the first bead of the returning sunlight, but shows some very pretty prominences on the west limb of the sun. It may be mentioned that the polar streamers around the sun's north pole are much more extensive than those around the south pole, which is the reverse of what was the case in the 1901 eclipse, and the north polar streamers are not symmetrical about the sun's axis.

A peculiarity upon which E. E. Barnard has remarked in connection with previous coronal photographs is again noticeable in the present corona, viz., a dissimilarity between the north and south polar fans: the north fan consisting of slender, sharply defined single streamers, whereas the southern is composed of broader, diffused streamers, occurring more or less in bunches.

This corona belonged to a type which has not been much studied, corresponding to an increasing sun-spot activity, following shortly upon an unusually protracted minimum. There are some resemblances both to the 1898 and to the 1901 coronas, but on the whole this corona differs from both of them.

(2) *Photographs of the corona through green colour filters.*

The aim of this portion of the programme was to photograph the corona through green colour filters, transmitting a narrow band of light in the region of the strong coronal line, the green line $\lambda 5303$, which is attributed to coronium, so as to obtain information as to the presence and distribution of coronium in the corona. The instruments used were (i.) the Astrographic Triplet—a Cooke triple objective of 6-inch aperture and 2 feet 3 inches focal length, and (ii.) the Simms photographic camera, with an object-glass also of 6-inch aperture, but with a focal length of 6 feet 6 inches. The colour screens used were a Wratten & Wainwright mercury monochromat and a Zeiss filter, which were practically identical as regards the intensity and spectral range of the transmitted light. In order to avoid misinterpretation of the results obtained, and to provide a means of discriminating between the continuous light from the corona which the filter might transmit and the monochromatic green light of wave-length $\lambda 5303$, a prism was placed before each object-glass to disperse the continuous light.

At the two points of the image, at the extremities of a diameter parallel to the refracting edge of the prism, the dispersion is tangential to the limb, so that there discrimination is impossible.

Accordingly, an arrangement was devised by means of which exposures could be made with the refracting edge of the prism, either horizontal or vertical as desired. For the purpose of changing from one position to the other, the telescopes were mounted so that their axes could move upon the surface of a right circular cone, the axis of which was in the direction of the beam of light reflected from the cœlostat. The two axes about which the telescopes rotated were fixed parallel to one another on a table supported by trestles, and the two telescopes were fed by one 16-inch cœlostat in all respects similar to that which fed the coronagraph. Four plates were exposed with each instrument, the times of exposure and the position of the refracting edge of the prism in each case being as follows:—

No.	Exposure.			Position of Refracting Edge of Prism.
	Beginning.	End.	Duration.	
	s	s	s	
1	2	17	15	Vertical.
2	20	60	40	,,
3	66	106	40	Horizontal.
4	109	124	15	,,

The exposures were made by the two observers simultaneously, and the positions of the telescopes were changed together, to avoid spoiling any exposures by jarring. The plates used were quarter-plate size, the four being carried in one plate-holder, so that no time was wasted in changing plates with either instrument. The plates used—Ilford No. 513—were specially prepared for us by Messrs. Ilford, Ltd., so as to be very sensitive in the green portion of the spectrum, whilst at the same time being insensitive in the red. They were found to be in all respects satisfactory. The presence of coronium in the corona should be indicated on the photographs by a sharp ring, superposed upon a fainter background due to the dispersed continuous light. The evidence which they afford, however, is negative. It is obvious from them that any results obtained without the use of prisms would have been of no value, and liable to serious misinterpretation. No trace of light from coronium is apparent, and as far as the evidence of these photographs goes, it appears that coronium, as indicated by the green line, was almost entirely absent from the present corona.* In a future eclipse, it would be well worth trying this same experiment again, but using a denser filter, transmitting a much narrower range of continuous spectrum, in combination with a prism of higher dispersion. The photographs obtained in the present eclipse will provide material by means of which one can estimate the times of exposure required.

* This conclusion is supported by the results obtained by the expedition to Strömsund from the Meudon Observatory (*Comptes Rendus*, 1914, Nov. 16). The spectrum of the corona does not show any trace of the usually strong green ray.

III. Photographs of the Spectrum of the Chromosphere and of the Corona.

The spectroscope employed was kindly lent by Professor Newall. It has been used in several previous eclipses, and was specially designed for use in the ultra-violet region of the spectrum, all the optical parts being of quartz. It has a train of four 60° prisms, each composed of two half prisms of right- and left-handed quartz. The height of the prisms is $2\frac{3}{4}$ inches and length of base $3\frac{1}{2}$ inches.

The objectives of the collimator and camera are single lenses of 3 inches aperture and 33 inches focus.

When employed at previous eclipses the solar image had been formed by a quartz lens supplied with light from a heliostat, with a silver on glass mirror. On the present occasion a concave speculum mirror of 9 inches aperture and 10 feet focus was substituted for the quartz lens to avoid the chromatic aberration of the latter, and a cœlostat with a speculum flat mirror was used in place of the heliostat. Speculum mirrors were used so that the spectrum might be extended as far as possible into the unexplored ultra-violet.

The concave mirror, fed from the cœlostat, reflected the beam back to a focus on the slit of the spectroscope just behind the cœlostat, the converging beam being made to pass as near as possible to the cœlostat to diminish the optical distortion. Although the angle between the incident and reflected rays was nearly 3°, the sun's image was well defined. The image on the slit was viewed from the side by means of a low-power microscope with a small reflection prism over its objective. A similar contrivance was applied to the end of the camera, so that the visual part of the spectrum might be watched whilst the ultra-violet was being photographed.

A part of the proposed programme was the photographing of the chromosphere at the cusps during the partial phase. The eyepiece at the camera-end was to enable the observer to control the exposure. He would there be able to see whether the spectral lines were bright or dark, and could stop the exposure when, through the sun drifting on to the slit, the bright lines disappeared.

To increase the length of spectrum in good focus, films carried in a curved sheath were used instead of plates as formerly. This was so far successful that critical definition was secured over a length of about 6 inches, the spectrum extending from λ 3000 to λ 4100. The dispersion varies considerably over this range, being large at the ultra-violet and small at the blue end. The following table shows this:—

At λ 3100	the dispersion is	4·5	tm. per mm.
3300	,,	6·7	,,
3500	,,	9·3	,,
3700	,,	12·1	,,
3900	,,	15·3	,,
4100	,,	18·8	,,

Arrangements were also devised for placing a spectrum of the iron spark on the photographs for the accurate determination of the wave-lengths of the coronal lines. Two exposures on the spark were to be given, one on each side of the coronal spectrum. Portable accumulators were taken with us to Russia, and charged at the electric light station in Minsk. From these, with the aid of an induction coil and a condenser, a strong spark was obtained.

The attempt to photograph the chromosphere at the cusps was not very successful. At the higher level only the brightest lines were visible, and at the lower level the sun boiled over into the slit, overpowering the chromospheric lines.

At 15 secs. before totality the programme for photographing the chromosphere at the "flash" was commenced. The disappearing limb was brought nearly tangential to the slit, and a series of exposures of 4 secs., the film being changed after each, were given by Mr. Hepburn, whilst Mr. Davidson with the slow motions of the cœlostat kept the sun's limb on the slit. The signal "Go" from Mr. Jones occurred between the last two exposures.

The film was again changed and the exposure on the corona commenced, the slit being kept tangential to the moon's limb. Just before the sun reappeared the exposure was stopped, having lasted about 120 secs. The iron-arc comparison spectrum was now placed on either side of the spectrum of the corona.

When the film was developed it was found that the "flash" had occurred between the last two exposures, so that the results were not quite so successful as had been hoped. However, the spectrum extends down to λ 3118, and 120 lines have been measured between this and λ 3300, the previous limit for accurate measurements of wave-length, and the definition over the whole range is very good.

The attempt to secure the spectrum of the corona was unsuccessful; probably the slit, narrowed to secure the finest definition in the chromospheric spectrum, was too narrow for the faint light of the corona. It may also be that the present corona was unusually faint in the ultra-violet region.

Total Solar Eclipse 1914 *August* 20–21.—*Report on an Expedition from the Solar Physics Observatory, Cambridge, to Theodosia, Crimea.* By H. F. Newall, F.R.S.

The Solar Physics Committee having sanctioned the sending of an expedition from the Solar Physics Observatory, Cambridge, to observe the total eclipse of the sun, 1914 August 20–21, it was arranged with the Joint Permanent Eclipse Committee of the Royal Society and the Royal Astronomical Society that the party should go to the Crimea. The expedition is indebted to the

Government Grant Committee of the Royal Society for a grant towards the expenses of an observer.

The choice of observing station was arranged by correspondence with Dr. Backlund and with Monsieur Bjeljawsky, the astronomer in charge of the Observatory at Simeis, and it was decided to occupy a station in the immediate neighbourhood of Theodosia. This town is a seaport near the east end of the south coast of the Crimea. It lies about 12 miles from the central line of totality, and the loss of three seconds in the duration of totality was likely to be more than compensated by the advantages afforded by the town in the accommodation of the observers.

The party, consisting of Professor and Mrs. Newall, Mr. Stratton, and Mr. C. P. Butler, travelled out to the Crimea *via* Trieste, the Mediterranean, and the Black Sea to Theodosia. Mr. R. Rossi, who had volunteered assistance in the preparations and observations, joined the party at Trieste.

The party arrived at their destination on July 25, in the early morning, and before midday the position of the camp was chosen among the vineyards on the slopes of the hill Pascha Tepe, about 4 versts (1 verst = 1·067 kilometre) to the north-west of Theodosia.

The instruments, which had been sent out direct from Hull to Theodosia, were found safely warehoused in the sheds of the Custom-house.

The next days were occupied in passing the instruments through the Customs and transporting them to the chosen site of the camp, in finding living quarters, and in arranging for the temporary furnishing of an empty bungalow which, by the kind intervention of Mr. Sarandinaki, the engineer of the port, had been generously put at the disposal of the party by Mr. Hadji, a native of Theodosia. Professor Perrine and Mr. Mulvey, of the Cordoba Observatory, with whom we travelled from Odessa to Theodosia, joined us in occupying the bungalow, and Messrs. Zurhellen and Freundlich of the Babelsberg Observatory found rooms in the neighbourhood and joined our mess.

Within four days of our arrival at Theodosia, red notices ordering the mobilisation of the Russian troops were posted up in the streets of the town, and four days later we received news of the declaration of war by Germany on Russia. The latter news reached us in the form of a notice from the Russian military authorities to the German astronomers to leave the country.

In these distracting and anxious circumstances, the preliminary preparations for the observations of the eclipse were made. The news of England's declaration of war on Germany was brought to us by the British Vice-Consul, Mr. von Stürler, at midnight within about 24 hours of the declaration. Mr. Stratton left us on August 11, to return to England *via* Odessa, Petrograd, and Norway to join the Officers' Training Corps at Cambridge.

All the necessary preparations for the observations were

completely made under considerable stress. The programme involved preparations for the following observations:—

1. Spectrographic observations of the "flash" and corona, to be made with a 6-inch Rowland grating of 21½ feet radius used in a camera of length 11 feet. (10 photographs.)
2. Polarigraphic observations of the corona with the Savart camera and comparison camera in connection with visual observations of the polarisation of the sky. (18 photographs.)
3. Polarigraphic observations of the corona with a large Nicol prism and camera. (4 photographs with the Nicol prism in four different settings at intervals of 45°, with standard squares for photometric purposes.)
4. Photographic observations of the corona with an enlarging photoheliograph giving a solar image about 1½ inches in diameter. (9 photographs with standard squares for photometric purposes.)
5. Spectropolarigraphic observations of the corona with a double-image spectrograph. (4 photographs, with two exposures.)
6. Photographic observations of the corona with Mr. F. McClean's 6-inch photovisual lens in a camera of focal length 31½ feet, kindly lent by Sir N. Lockyer. (5 photographs for special study of light and dark arches in the corona.)

The whole equipment was covered under two roofs, a framed shed and a double-walled hospital marquee covering a ground plan 30 feet by 16 feet. The latter marquee was found to be a valuable provision against the heavy thunderstorms and the hot sun. The two covering tents were specially oriented in a way that gave successful and economical conditions for assembling the apparatus for the required observations.

Drills for practice in the operations required for the eclipse observations were begun on Tuesday, August 18, and in this work we had welcome and efficient help from Captain Stevens and Captain Morgan and their officers from two British merchant ships lying in the harbour of Theodosia. The drills were continued on the afternoons of Wednesday and Thursday, and all those who took part quickly became proficient in the help they had so kindly volunteered to give in the eclipse operations.

In relatively early days of the preparations we had realised that the chances of successful observations depended entirely on the direction of the wind. With the wind from the south-west cumulus clouds would rise about midday over the region of Stary Krim, a considerable hill lying at a distance of about 25 kilometres, nearly in the same azimuth as the sun at the hour of totality, and for the rest of the afternoon they would be driven in more or less broken masses over the sky. Sometimes they were the harbingers

of heavy thunderstorms. With the wind in other quarters we would have brilliant blue skies, which augured well for exceptional conditions for eclipse observations.

On the day of the eclipse, Friday, August 21, the wind was from the south-west. After a fine morning, which was spent in the final operations of the adjustments, broken clouds began to come up from Stary Krim and float across the sky in increasing numbers. Our only hope lay in the unlikely chance that the total phase would be visible in a small patch of blue sky between the masses of cloud.

About one minute before totality a thick detached cloud drifted up relentlessly in front of the eclipsed sun and completely hid the whole of the total phase from our view. One or two glimpses of the reappearing crescent were caught through thinner parts of the obscuring cloud, but it was not till three or four minutes after the third contact that the cloud had drifted past the sun's position and disclosed the growing crescent in blue sky.

All our preparations were in vain. It was not possible to secure a single photograph during the total phase of the eclipse.

Our disappointment was shared by Mr. Perrine, who had set up his fine installation of instruments close to our camp. MM. Bjeljawsky and Neujmin had made their camp about one verst nearer to Theodosia, and the cloud, which had completely obscured our view of the eclipse, reached them a few moments later; they were able to get observations for about 30 seconds, and then their view was obscured. Signor Riccò, who was stationed in the town, secured observations throughout the whole of the total phase. The same good fortune attended the efforts of the Count de la Baume Pluvinel, Dr. Donitch, Prof. Sternberg, M. Chrétien, and M. Ascarza.

The experience may be taken as showing the importance of spreading the camps of observing parties even in the restricted area chosen as a station of observation.

The expedition was much indebted for assistance received in many varied ways:—To the Russian Government for facilities in respect of Custom duties and the warehousing of the cases of instruments; to Dr. Backlund and Monsieur Bjeljawsky for much helpful assistance in preliminary inquiries; to Mr. Sarandinaki, the engineer of the port of Theodosia, who was in charge of the Meteorological Station, for great help in practical matters concerning the finding and establishing our living quarters; to Mr. von Stürler, the British Vice-Consul, for many helpful kindnesses; to Mr. and Mrs. Lemke, for kind help in household preparations; to Dr. Krim for permission to use his land for our camp among the vineyards; to Mr. Hadji for the loan of a summer bungalow within a couple of hundred yards of the instruments; to Captain Stevens and Captain Morgan and their officers from two British merchant ships lying in the harbour of Theodosia, for their willing and excellent services in the drills and help in the operations needed for the observations.

Mr. Stratton returned to England on August 11, and the rest of the party left Theodosia on August 27. Mr. Rossi went to Odessa to make his way back to Italy. Professor and Mrs. Newall and Mr. Butler returned to England *via* Simeis, Sebastopol, Moscow, Petrograd, Raumo, Gefle, Stockholm, and Bergen, reaching England on September 17.

The instruments are stored at Odessa in charge of Messrs. Jacob Owen & Co., agents of the Wilson Line.

Reply to Professor Sampson's Objections to the Hypothesis of a Sun-spot Swarm. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. My reply to Professor Sampson's objections might be compressed within the limits of a single sentence—he has omitted to take any account of the difference between a small compact planet and a large extended meteor-swarm. Expanding this sentence a little, he has found a prohibitive mass for the Leonids on the assumption that they can only act as a massive particle would act; and he has dealt with their orbit as if it were as well defined, and as well known, as a planetary orbit. His objections are thus irrelevant, and I might well call upon him to substantiate his sweeping judgment, that "it seems quite certain that nothing so striking as Professor Turner's theory requires can remain to be discovered," by something more relevant. He claims that he is "not concerned to defend the precise form of a compressed note inserted upon the proof-sheets of a work which is itself a mere summary": but he does not explicitly withdraw this judgment:* and, moreover, he goes on to profess concern lest my theory "should find a place in astronomical literature without receiving careful examination." I cordially share his concern. But is it not of equal importance that a hasty condemnation of this theory, based upon no examination at all, should "find its place in astronomical literature"? Against the former risk I took some pains: I cannot congratulate Professor Sampson on his precautions against the latter. In order to supplement them a little I here submit a brief examination of his chief points.

2. First, as regards the objection that to produce the requisite perturbations in the SSS, the Leonids must have a prohibitive mass. Professor Sampson considers "as a definite problem the encounter of a small body by a planet," and finds that to perturb the orbit of the "small body" the "planet" must have a mass

* I can scarcely pass without comment Professor Sampson's claim that "the statements of fact . . . are clearly distinct from statements of [his] own judgment upon them." The two sets of statements are not actually superimposed, which would be difficult, but they are juxtaposed as closely and as significantly as possible.

comparable with that of Jupiter. Without challenging or examining his solution in any way, I remark that it is entirely defective as a comment on the action of a long stream of meteors, because the sphere of action in his selected case is considerably restricted. He calculates that for a planet P with mass three times that of the Earth the radius of the sphere of action is $d = 0{\cdot}03$ in astronomical units. As P travels in its orbit round the Sun, this sphere of action is carried with it, forming an elongated region within which action is possible at suitable times. But though this region has one long dimension, the perturbed body must necessarily cross it rapidly. It could only remain in the region to good purpose by traversing the long dimension and also travelling with the same velocity as P, a case which Professor Sampson has reasonably disregarded (end of his seventh paragraph).

3. When, however, we are dealing with a long stream of meteors the case is essentially altered. We know the stream in question to be long, because we get showers of Leonids in successive years. A length of 10 units is not unreasonable. The region of action at any given moment now resembles a long narrow cylinder, rounded at the ends; as the stream travels this cylinder sweeps out a region which will have a sensible breadth, as well as length, unless it is travelling accurately in the direction of its length. It is easy to assume that the whole stream is following the same orbit, so that the cylinder would travel in the direction of its length: but are there any grounds for such an assumption? Is it not, indeed, far more probable that the orbits of different members of the stream are different? For whatever cause acted differentially upon them to extend them into a stream would act differentially in other ways, unless reason can be given for unity in one item and diversity in others. Professor Sampson has in any case advanced nothing to this purpose; and I am entitled to ask him why, instead of increasing the mass P, I may not distribute the Leonid stream at a slight inclination to its path round the Sun?

4. An inclination of 1 in 10 would then give the region of action a breadth of 1 unit: so that if the SSS enters this region on one side and leaves it on the other, it must have been removed *at least* 1 unit in the meantime. It will make such an entrance and exit if it encounters the stream near one end and remains within its influence until it reaches the other end. For, regarding the SSS as at rest and the stream as travelling with velocities 10 V in the direction of its length and V transversely, we can (so far as relative motion is concerned) attribute these velocities (reversed) to the SSS. Clearly the large velocity 10 V will then carry the SSS from end to end of the stream, provided the transverse conditions are satisfied: that is, provided the transverse attraction is sufficient to retain the SSS within the cylinder of action. It is not meant that this velocity 10 V will remain constant: for it is subject to modification by the Sun's attraction and by that of the Leonids. By hypothesis (*i.e.* since we are

within the region of action) the latter will be the greater, and will tend towards the centre of the swarm. At first therefore it will increase 10 V: later it will combine with the Sun's attraction to diminish it: but we need not spend time in showing that 10 V cannot be reduced to zero, since the whole question is whether the much smaller V can be reduced to zero by the attraction of the swarm, so as to retain the SSS within the region of action.

5. Hence the SSS will traverse the stream from end to end if it can be retained laterally. Now there is one case in which it can clearly be retained, viz. if it enters the region of action with no lateral velocity. This seems to be inconsistent with what has just been said about the lateral velocity being V: but that was said on the assumption that the SSS was absolutely at rest at the beginning of the encounter. There is no reason why it should not have a velocity comparable with V and yet be quite near aphelion. We have from the conditions of motion of the Leonids

$$(10\mathrm{V})^2 = 1{\cdot}6\mathrm{S}/10 = \mathrm{S}/6$$

where S is the Sun's mass. For the SSS

$$v^2 = \mathrm{S}\left(\frac{2}{r} - \frac{1}{a}\right) = 600\mathrm{V}^2\left(\frac{2}{r} - \frac{1}{a}\right).$$

We may take $a = 5$ and $r = 10$ at aphelion. For other values of r the values of v/V are as follows:—

$r =$	10·0	9·94	9·90	9·80	9·70
$v/\mathrm{V} =$	0·00	0·85	1·10	1·57	1·93

6. Hence v_1 resolved at a finite angle may quite close to aphelion of the SSS be equal to V, in which case the SSS will enter the region of action with no transverse velocity. Since the transverse attraction is then by hypothesis greater than that of the Sun, the SSS will remain within the region until it has traversed it from end to end. The only condition to be fulfilled is that a region of action of reasonable dimensions should exist. For a planet three times the mass of the Earth Professor Sampson finds the distance of action to be 0·03. Let us imagine n^2 equal planets of radius $5/n^2$ placed in a row so as to make a stream 10 units in length, and of total mass equal to the above planet. The mass of each being then $1/n^2$ of the former, the distance of action will be $0{\cdot}03/n$ and that of the whole stream will naturally be greater, in a ratio which we will disregard for the moment. We can suppose each individual planet to fill up its own sphere of action without altering the limit, in which case

$$0{\cdot}03/n = 5/n^2$$

or $$n = 167.$$

Hence each radius $0{\cdot}03/n$ is about 17,000 miles, and the width of the stream 34,000 miles. If the density is uniform throughout each sphere, it will be $3/n^2 \times 4^3$, or about 2×10^{-6}, that of the

Earth—say the equivalent of a rock 60 feet cube, or 30,000 tons, in each cubic mile.

7. The Earth, travelling at 1000 miles a minute, would cross such a stream at right angles in 34 minutes, and obliquely in a few hours. But if there is such a stream, the Earth certainly does not encounter the core of it, or we should know it to our cost. The core, however, to which the above diameter may be taken to refer approximately, would quite naturally be surrounded by matter of decreasing density, and all the phenomena we know would be satisfied by the passage of the Earth through these somewhat distant surroundings. Indeed, to encounter so narrow a core as that above assigned would require an extraordinary coincidence, only to be explained in the case of the SSS by the postulated community of origin. The possible encounters with Saturn must be regarded as a coincidence, though, owing to the greater size of Saturn, and possibly also to the effect of his attraction, the coincidence need not be so striking as it would be in the case of the Earth.

8. The above figures are not intended to be definitive: they are simply intended to show that Professor Sampson's calculations leading to a prohibitive mass are far from exhaustive or conclusive; in fact, that he has not dealt with the actual problem at all, but with an irrelevant one.

9. I come now to his treatment of the orbit. I do not think he realises how indefinite our knowledge of the orbit is. We may be able to assign a fairly definite orbit for the few Leonids which strike the Earth, but even then we cannot identify the portion we meet on one occasion with that we meet on another. And I cannot suppose he means to deny the existence of the rest of the swarm because we never encounter it. We know it to have considerable length, and it is natural to assign it reasonable breadth. It has been urged above that there is nothing unreasonable in lateral dimensions of one unit. Hence Professor Sampson's revised calculations which bring Saturn within half a unit of the orbit give quite enough latitude for an encounter even with the central mass, which may well be at this distance from the outlying portion encountered by us. The fact that the perturbations calculated by Adams and others for the part known to us agree with their observed values merely proves that these investigators have done the arithmetic correctly; it is quite irrelevant to the existence of an associated portion of which we have hitherto had no evidence, unless indeed Sun-spots have been trying to bring it to our notice in just the manner I now suggest. Even the distance of Saturn in 1863 from the orbit of the portion known to us, large as it is, is no *fatal* objection (such as Professor Sampson undertook to establish) to the existence of an associated portion following this different route. I made the suggestion of such an encounter in 1863 with my eyes open to these requirements; and I took the particulars of the known orbit from a diagram, because I considered that a rude approximation was sufficient. Even the fact that

the diagram was erroneous does not matter, so long as there is a *possibility* that the swarm is widely diffused. It is for those who deny the *possibility* to make good their contention.

10. One correction I have to make. When Professor Sampson read his paper, I said that his dealing with Leverrier's date A.D. 126 for the introduction of the Leonids into their present orbit was irrelevant. I was betrayed into this remark by a momentary anxiety to have five minutes to reply to the objections already raised, and I now withdraw it absolutely, and substitute the cordial agreement that this part of Professor Sampson's paper is of the very greatest importance, not only from the general point of view, but from that of my own selfish needs. His able analysis points to the conclusion that A.D. 885, and not A.D. 126, is the probable date of the introduction into their present orbit of the Leonids as we know them — may I say L_0? Then first I remark that this does not preclude the introduction into a very similar orbit, also by the action of Uranus at another date, of an associated swarm L_x. Consider the circumstances and let us accept Professor Sampson's version. It follows that there was in A.D. 885 a swarm of meteors travelling in some orbit O_1, which crossed the orbit of Uranus; and that Uranus was so close to a portion of the swarm in A.D. 885 that it diverted a portion of it, L_0, into the orbit O_2, well known to us. Is it more likely that it diverted the *whole* swarm or a *portion* of it? At any rate I will put in a claim that it was a portion only, the rest continuing to travel in orbit O_1: so that at some later date Uranus might get a chance to divert more of them. But I want to invert history and look backward to an earlier date instead of forward to a later. It is also possible that when the swarm was larger, Uranus got an earlier chance—say near A.D. 126 with Leverrier; we cannot say until we know the period of orbit O_1—which produced L_x travelling in an orbit very like O_2, because started under conditions very similar to those for L_0. Is there anything impossible in this? It may strike one as *unlikely*, because it involves a nice adjustment between three periods, but my claim is that the mass of evidence collected in my former paper makes such points worth investigating. Now I call Professor Sampson's work on Leverrier's date specially valuable, because it may enable us to determine orbit O_1 and get its period. This done, we could search for other possible encounters between Uranus and the swarm O_1, and see whether we get a date near A.D. 188.

11. I am well aware that this is only sketching possibilities instead of testing them, and that such sketches as I have given carry with them responsibilities to the discharge of which nothing has yet been done. To avoid misunderstanding, may I state one or two facts? My former paper was completed rather hurriedly, because I was faced with some lectures which required a good deal of preparation. Owing to the illness of one friend and the death of another, I have, during the past year, had to pass through the press a volume of *Transactions*, and to superintend the Seismo-

logical work at Shide; and the visit of the British Association to Australia took up several months. These extra calls on one's time might perhaps have been avoided: it is sometimes difficult to choose between conflicting duties. But I frankly acknowledge that a greater difficulty has been to decide where first to get hold of an end of this tangled skein. My opinion hitherto has been in favour of Saturn's orbit. We have the definite testimony of the great man we have just lost, G. W. Hill, that the modern observations show some unknown disturbance, and I think that to discuss the residuals since 1863 might throw light on the problem. I was stopped by the gap between 1888 and 1900, and it was only just before leaving for Australia, and after having had special computations made needlessly, that I learnt that Hill himself had filled this gap. His work makes the problem fairly straightforward; but my experience of such work is limited, and the arrears resulting from a long voyage are not easily subdued. Skilled help would be very welcome, and none more than that of Professor Sampson, if he is able to give it.

A Proposal for the Comparison of the Stellar Magnitude Scales of the Observatories taking part in the Astrographic Catalogue. Fifth Note. The Perth (W.A.) Magnitudes. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The Perth (West Australia) results for zone -32° have been published in four volumes, of which the first appeared some little time ago, the last during the present summer. An examination of the first volume suggested that we had at last a case where the galaxy was richer in photographically faint stars, as announced by Kapteyn for visually faint stars generally. All the material examined in the northern hemisphere had presented a conclusion directly at variance with this; and when the next volume of Perth results appeared, giving results for the other region where the galaxy crosses zone -32°, Mr. R. J. Pocock made a careful analysis of it, with results shown in the next paper (p. 148) and confirmatory of the exceptional case. Comments will be made on this point after the presentation below of the results for the whole zone rendered possible by the appearance of the remaining two volumes. In this presentation Mr. Pocock's work has provided the results for $6^h - 12^h$, and I tender him my thanks for permission to use them.

2. An hour of R.A. of zone -32° contains either six or seven plates, these numbers alternating regularly. To secure uniformity, the results were all multiplied by 10 and then divided by 6 or 7, so as to form uniform averages for 10 plates, which are given in Table I. In the first column is given the hour of R.A.: in the second the total number of bright stars, for which measured diameters are given in the Catalogues. No detailed analysis of

these measures has been attempted, as the number of stars is small. Then follow columns for the classes A to M, designated by these letters in the Catalogues; these classes being determined by comparison with a prepared scale of images. The last column gives the totals for each hour. At the foot of the table are given the totals for each column; the cumulative totals below (N′) and then

$$\log N_0' = \log N' - \log 240 \; [= 2{\cdot}380]$$

is the logarithm of the cumulative total for a single plate, each column representing 240 plates altogether. If we assume that the whole zone is a fair sample of the whole sky (an assumption which may need a sensible correction to the magnitude zero-point), we can then determine from log N_0' the photographic magnitude on the scale of Chapman and Melotte, which is the best of the kind we have yet had available.

TABLE I.

Perth Star Counts for 10 *Plates.*

R.A. h	Brt.	A.	B.	C.	D.	E.	F.	G.	H.	J.	K.	L.	M.	Total
0	13	10	40	41	60	89	146	179	222	264	168	56	3	1291
1	13	16	25	36	44	66	138	170	215	212	125	47	2	1109
2	10	8	22	25	42	114	130	153	267	278	207	69	0	1325
3	9	16	34	34	39	102	154	181	236	223	139	31	1	1199
4	6	16	16	36	39	85	151	189	286	397	189	96	4	1516
5	15	24	10	49	67	130	200	234	315	495	357	180	70	2146
6	34	34	31	86	105	175	308	494	594	706	584	280	16	3447
7	35	56	58	145	177	296	569	958	1037	1024	788	650	13	5803
8	35	68	69	157	198	509	773	1203	1643	2057	1813	720	103	9347
9	26	49	69	113	133	293	499	800	951	989	874	433	1	5231
10	21	36	66	90	130	170	340	575	592	688	764	236	0	3708
11	30	28	27	75	122	197	305	524	508	585	455	250	0	3106
12	15	29	37	78	65	145	245	424	408	408	448	260	0	2563
13	34	21	27	61	84	184	284	451	534	564	490	303	0	3037
14	31	34	34	80	96	238	360	631	763	720	715	656	0	4358
15	39	29	36	88	101	185	398	581	680	785	620	487	9	4038
16	42	60	47	140	124	236	416	686	1064	1129	1100	786	283	6113
17	61	91	110	231	333	609	944	1612	2193	2007	1803	2080	184	12255
18	49	73	80	135	167	385	629	1150	1649	2247	1934	540	83	9122
19	13	44	56	93	89	180	286	445	719	1150	804	113	13	4000
20	10	20	24	50	67	128	232	312	502	800	581	135	17	2877
21	9	9	20	45	35	58	82	162	258	365	273	75	22	1411
22	6	9	12	29	29	82	84	149	217	240	154	31	11	1053
23	17	15	28	53	80	175	252	265	275	315	181	20	0	1676
Total	573	795	978	1970	2426	4831	7925	12528	16128	18648	15566	8534	835	91737
N′	573	1368	2346	4316	6742	11573	19498	32026	48154	66802	82368	90902	91737	
log N_0'	0·378	0·757	0·991	1·255	1·449	1·683	1·910	2·125	2·303	2·445	2·536	2·579	2·583	
Mag. C. & M.	7·78	8·55	9·05	9·64	10·08	10·64	11·20	11·76	12·25	12·65	12·91	13·06	13·08	

3. A small liberty has been taken with this scale, which its authors may not approve. They have used a rather complex formula for its construction, and it is not intended here to question in any way the assumptions made in that construction. But for practical purposes and within the limits indicated in Table II., the formula indicated in Table IX. of their paper (*Mem. R.A.S.*, vol. lx.) can be represented by the formula

$$\log B_m = a + 0{\cdot}400(m - 11) - 0{\cdot}0160(m - 11)^2$$

where m is the star's magnitude, and a is a constant which depends on the area selected as standard. Chapman and Melotte use for this purpose the whole sky: I have substituted as more convenient for the purposes of the *Astrographic Catalogue* an area of 4 square degrees, which is approximately that of a single plate. We thus subtract from their constant the quantity

$$\log 4 \times 180^2/\pi - \log 4 = 4{\cdot}013.$$

The following expanded table may be found useful:—

TABLE II.

Chapman and Melotte's Scale of log N for four square degrees.

Mag.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9
6·0	9·430	9·486	9·542	9·597	9·651	9·706	9·760	9·814	9·868	9·921
7·0	9·974	0·027	0·079	0·131	0·183	0·234	0·285	0·336	0·386	0·436
8·0	0·486	0·535	0·585	0·633	0·682	0·730	0·778	0·825	0·872	0·919
9·0	0·966	1·012	1·058	1·104	1·149	1·194	1·239	1·283	1·327	1·371
10·0	1·414	1·457	1·500	1·542	1·584	1·626	1·667	1·708	1·749	1·790
11·0	1·830	1·870	1·909	1·949	1·987	2·026	2·064	2·102	2·140	2·177
12·0	2·214	2·251	2·287	2·323	2·359	2·394	2·429	2·464	2·498	2·532
13·0	2·566	2·589	2·632	2·665	2·698	2·730	2·762	2·793	2·824	2·855
14·0	2·886	2·916	2·946	2·975	3·005	3·034	3·063	3·091	3·119	3·147
15·0	3·174	3·201	3·228	3·254	3·280	3·306	3·331	3·356	3·381	3·406
16·0	3·430	3·454	3·478	3·501	3·523	3·546	3·568	3·590	3·612	3·633

4. Comparison with this scale shows that the Perth adopted scale is remarkably uniform.

TABLE III.

Comparison of Perth Classes with a Uniform Scale.

	A.	B.	C.	D.	E.	F.	G.	H.	J.	K.	L.	M.
O	8·55	9·05	9·64	10·08	10·64	11·20	11·76	12·25	12·65	12·91	13·06	13·08
C	8·56	9·08	9·60	10·12	10·64	11·16	11·68	12·20	12·72	13·24	13·76	14·28
O − C	− ·01	− ·03	+ ·04	− ·04	·00	+ ·04	+ ·08	+ ·05	− ·07	− ·33	− ·70	− 1·20

The interval is apparently about $0^m{\cdot}52$,* and as far as class J the largest residual is + ·08 for G. After this the last three classes should apparently count as one, class M following class J. The residual would then be 13·08 − 13·24 = − 0·16.

5. We now consider the distribution in R.A. and follow

* In the printed volumes of Perth results this interval is estimated at 0·3 only, and is accordingly so mentioned by Mr. Pocock.

former procedure in calculating the harmonics for different magnitudes. It will suffice to discuss points separated by whole magnitudes, and we select the points represented by B D F H M. The brighter stars are scarcely numerous enough to give trustworthy results from a single zone. In Table IV. residuals are given in the body of the table, following the practice initiated in the last paper ("Cape Results," *Mon. Not.*, lxxv. p. 63). To recover any original constituent we must add the first number in the row (for B stars) and the constant at the foot of the column. Thus for M Stars in R.A. 0^h the original constituent log N′ was $(348-25-12)/100=3{\cdot}11$. In the last column the difference H − D is formed to show the effect of excluding the faintest stars and reducing the effect of the brightest: it will be seen that the changes of sign are very similar to those in M alone (preceding column).

TABLE IV.

Differences log $N'-N_B'$ for each Hour of R.A. in Units of 0·01.

R.A.	Gal. Lat.	B.	D.	F.	H.	M.	H − D.
h	°						
0	−85	−12	− 5	−11	−19	−25	−14
1	−78	−19	− 6	−11	−16	−24	−10
2	−66	−32	− 3	+ 4	0	− 4	+ 3
3	−54	−15	−11	− 9	−15	−25	− 4
4	−40	−34	+ 1	+ 5	+ 5	+ 4	+ 4
5	−28	−23	+ 7	+ 9	+ 4	+ 8	− 3
6	−16	+ 8	0	− 2	− 2	− 2	− 2
7	− 4	+25	+ 4	+ 5	+ 6	+ 3	+ 2
8	+ 6	+32	+ 2	+11	+14	+17	+12
9	+15	+24	− 3	0	+ 2	0	+ 5
10	+22	+17	− 1	− 7	− 7	− 8	− 6
11	+28	+ 1	+ 6	+ 5	+ 4	0	− 2
12	+30	− 1	− 2	− 3	− 4	− 6	− 2
13	+29	− 1	− 1	+ 2	+ 3	+ 1	+ 4
14	+25	+ 8	− 2	+ 3	+ 7	+ 8	+ 9
15	+18	+10	− 1	+ 1	+ 2	+ 3	+ 3
16	+10	+25	− 1	− 6	− 1	+ 6	0
17	0	+50	+ 4	+ 5	+ 8	+11	+ 4
18	−12	+38	− 6	− 3	+ 5	+10	+11
19	−24	+13	− 4	− 8	− 6	− 1	− 2
20	−36	−19	+ 4	+ 9	+11	+17	+ 7
21	−48	−34	+ 3	− 8	− 4	+ 1	− 7
22	−62	−49	+ 4	+ 6	+ 7	+ 3	+ 3
23	−74	−14	+ 5	+10	− 1	−11	− 6
	Const.	192	238	283	321	348	83

6. The Galactic Latitude is given in the second column, taken from Kapteyn's table in *Gron. Pub.*, No. 18, for the middle of each hour.

TABLE V.

Differences M–B arranged in Galactic Latitude.

Gal. Lat.	B.	M–B.	Gal. Lat.	B.	M–B.	Gal. Lat.	B.	M–B.	Gal. Lat.	B.	M–B.
−85	−12	−25	−48	−34	+ 1	−12	+38	+10	+18	+10	+3
−78	−19	−24	−40	−34	+ 4	− 4	+25	+ 3	+22	+17	−8
−74	−14	−11	−36	−19	+17	0	+50	+11	+25	+ 8	+8
−66	−32	− 4	−28	−23	+ 8	+ 6	+32	+17	+28	+ 1	0
−62	−49	+ 3	−24	+13	− 1	+10	+25	+ 6	+29	− 1	+1
−54	−15	−25	−16	+ 8	− 2	+15	+24	0	+30	− 1	−6
Mean −70	−24	−14	−32	−15	+ 4	(± 8)	+32	+ 8	+25	+ 6	0

If we arrange the numbers B (representing totals) and the residuals M or M–B, representing excesses of faint stars, according to Galactic Latitude, as in Table V., it is seen that there is a sensible amount of variation in M–B with Galactic Latitude, but that there are other features in both B and M–B. For instance, the smallest values of B are by no means near the Galactic Pole: the mean value from Galactic Latitude -66° to -28° is sensibly smaller than for -73° to -85°. There must be other phenomena present, and probably more important phenomena than variation with Galactic Latitude. The existence of important differences between different regions of the sky, quite independent of Galactic Latitude, has been indicated by other results, especially those for Bordeaux and the Cape; and it seems better to await the discussion of these other phenomena, and perhaps their elimination from the results, before studying the minor effect (if any) of Galactic Latitude.

7. Meanwhile we give the harmonics for these numbers, as in previous papers; referred to $0^h{\cdot}0$ as epoch (it should have been made clear in former papers that the $0^h{\cdot}5$ introduced in counting has been definitely removed from the phase of the harmonics) we get the following figures, the unit being 0·01 as before :—

	Mag. diff.	
D − B	1·02	$0{\cdot}6\cos(\theta - 191^\circ) + 2{\cdot}4\cos 2(\theta - 133^\circ)$
F − B	1·96	$1{\cdot}6\cos(\theta - 145^\circ) + 2{\cdot}1\cos 2(\theta - 112^\circ)$
H − B	3·19	$4{\cdot}7\cos(\theta - 206^\circ) + 4{\cdot}8\cos 2(\theta - 107^\circ)$
M − B	4·02	$7{\cdot}7\cos(\theta - 223^\circ) + 9{\cdot}0\cos 2(\theta - 106^\circ)$

The maximum for the first harmonic thus falls near $14^h{\cdot}5$ in Galactic Latitude $+25^\circ$, and the minimum near $3^h{\cdot}5$ in Galactic Latitude -54°. The maxima for the second harmonic fall near $7^h{\cdot}0$ and $19^h{\cdot}0$, which are reasonably near the points where the Galaxy cuts the zone (viz., $7^h{\cdot}5$ and $17^h{\cdot}0$). The further discus-

sion of these numbers is reserved until other zones have been presented, but it may be remarked that the change from negative to positive occurs abruptly between 3^h and 4^h and the reverse change between 22^h and 23^h. The mean values are :—

	D.	F.	H.	M.
4^h to 22^h	+1	+1	+ 3	+ 4
23^h to 3^h	−4	−3	−10	−18

The Number of Stars of Different Magnitudes in the Perth Astrographic Catalogue, Vol. II. By R. J. Pocock.

[In consequence of the result arrived at by counts on vol. iv., quoted above, I suggested that Mr. Pocock should make an independent examination of vol. ii. This paper was written nearly a year ago, but as the remainder of the zone was published soon afterwards, it seemed desirable to present the whole material together, for convenience of reference. Comments are made on this paper in the preceding paper.—H. H. Turner.]

This volume comprises plates with centres in R.A. $6^h\ 0^m$ to $11^h\ 51^m$, Declination − 32°, crossing the Milky Way between about 7^h and 8^h R.A. Considering first the total number of stars measured on the different plates, we find that the star density rises to two maxima at approximately $7^h\ 21^m$ and $8^h\ 15^m$, with a low minimum between. After the second maximum there is a gradual and uniform descent; the densities between 6^h and 7^h, at the minimum about $7^h\ 40^m$, and again at about 10^h being approximately equal. Table I. shows the mean number of stars per plate for different groups of plates.

Table I.

Mean Number of Stars per Plate in Different Regions.

Limits of R.A.	Mean No. of Stars.	Logarithm.	Limits of R.A.	Mean No. of Stars.	Logarithm.
h m h m			h m h m		
6 0 to 7 3	356	2·55	8 42 to 9 18	652	2·81
7 12 ,, 7 21	911	2·96	9 27 ,, 10 3	422	2·63
7 30 ,, 7 57	451	2·65	10 12 ,, 11 6	377	2·58
8 6 ,, 8 33	1056	3·02	11 15 ,, 11 51	299	2·48

The plates have been grouped together, not for the purpose of smoothing irregularities—which indeed are small,—but for brevity. We next inquire whether there is any difference in the relative proportion of bright and faint stars depending on the total star density.

In the Perth Catalogue the magnitudes are denoted, not by measured diameters—except in the case of the brightest stars—

but by letters, corresponding to a scale in the eyepiece of the measuring instrument, the letters running from A to M, the difference between two consecutive letters being approximately ·3 of a magnitude. Let N_X denote the total number of stars on any plate not fainter than "magnitude" X. Thus N_M will denote the total number of stars on the plate. We now consider the values of $\log N_M - \log N_X$ for the different plates, giving X its different values A, B, . . . M in turn.

On drawing the curve of this function we see at once that for early values of X it closely resembles the curve of $\log N_M$, with the two maxima at the corresponding points. These become less marked, until by the time we reach X = K only one is visible and that not very conspicuously. As an example we take X = E and group the plates as before.

TABLE II.

Mean Values of $\log N_M - \log N_E$ *for Groups of Plates.*

Limits of R.A.	Mean Values of $\log N_M - \log N_E$.	Limits of R.A.	Mean Values of $\log N_M - \log N_E$.
h m h m		h m h m	
6 0 to 7 3	0·85	8 42 to 9 18	0·89
7 12 ,, 7 21	0·97	9 27 ,, 10 3	0·89
7 30 ,, 7 57	0·84	10 12 ,, 11 6	0·86
8 6 ,, 8 33	0·97	11 15 ,, 11 51	0·81

This indicates that the relative number of faint stars increases with the total star density. This is the result found by Kapteyn.* Previous counts of volumes of the *Astrographic Catalogue* in the northern hemisphere had not confirmed Kapteyn's conclusion; but the only other volume of the southern hemisphere so far counted (*Perth Astro. Cat.*, vol. iv.) gave the same result as above.

There is no doubt that the results from individual plates may be affected by accidental errors, due to the difference in the night, plate, etc. For example, comparing with the Cape Photographic Durchmusterung, we find that the mean of the magnitudes given there for the stars recorded as D in the Perth Catalogue is about 9·2 on the plate with centre at $7^h\ 3^m$, and only 8·9 on the plate, centre $7^h\ 12^m$. (The approximate value given in the Perth introduction is 9·3.) I have tried to adjust the figures so as to compensate for such errors, but have not had time to do so satisfactorily. No such adjustment, however, seems likely to do more than modify the above results; whether the plates are taken individually or in groups the phenomena is too marked and too persistent to be anything but real.

* Publications of the Astronomical Laboratory at Groningen. No. 18, "On the Number of Stars of Determined Magnitude and Determined Galactic Latitude," by Professor J. C. Kapteyn.

On the Determination of Fundamental Photographic Magnitudes.

By J. Halm, Ph.D. (Plate 13.)

In a paper entitled "A System of Photographic Magnitudes for Southern Stars" * reference was made to the fact, pointed out by Schwarzschild and others, that if light of different intensities I_1 and I_2 be exposed on a plate during the time-intervals t_1 and t_2 so that $I_1 t_1 = I_2 t_2$, the assumption of strict equality of the resulting actinic effects is not justified. Hence the so-called "law of reciprocity," according to which identical actions It produce the same actinic results, does not appear to be strictly applicable to photometric investigations based on the effect of light on photographic plates. No doubt the appreciable deviations from this law and the absence of a precise knowledge of their character have been partly responsible for the uncertainty attaching to astrophotometric results derived from the photographic method. This fact was recognised by Schwarzschild at the very outset of his important researches.† We owe to him one of the first and most fruitful experimental investigations into the character of the relation between the action of the light-source and the actinic effect produced on the plate, as well as a fundamental application of his experimental results to the problem of the photographic magnitudes of stars.

The modification of the law of reciprocity, at which Schwarzschild arrived, may be expressed as follows: Two sources of intensities I_1 and I_2 produce equal actinic effects, if the times of exposure t_1 and t_2 are so chosen that

$$I_1 t_1^{p} = I_2 t_2^{p}, \qquad . \quad . \quad . \quad . \quad (1)$$

p denoting an exponent sensibly constant, *i.e.* independent of the intensity, within the range of Schwarzschild's experiments. Equation (1) may also be written in the equivalent form:

$$I_1^{q} t_1 = I_2^{q} t_2, \qquad . \quad . \quad . \quad . \quad (2)$$

where $pq = 1$.

The correctness of the "Schwarzschild law," as formulated by equation (1) or (2), was accepted in the determination of the photographic magnitudes of the south polar area published in my previous paper. Subsequent to this publication, however, I received a letter from Professor Eberhard of the Potsdam Observatory, in which he drew my attention to an experimental investigation made by Dr. Kron ‡ with the view of testing the

* *Monthly Notices*, lxxiv. p. 600.

† *Publikationen der v. Kuffner'schen Sternwarte Wien-Ottakring*, Band v.

‡ *Publikationen des Astrophysikalischen Observatoriums zu Potsdam*, No. 67: "Ueber das Schwaerzungsgesetz photographischer Platten."

constancy of the exponent q over a wider range of intensities, and also with reference to plates of different degrees of sensitiveness. The most important result of Dr. Kron's work is undoubtedly the establishment of the "Schwarzschild law" as a *special* representation of a more complicated relation between intensities and times of exposure, which is approximately correct only for great and small intensities. This limitation of the applicability of the Schwarzschild law necessitates a re-examination, in the light of Dr. Kron's experimental evidence, of the magnitudes obtained by the method advocated in my previous paper. It is, therefore, the object of the present communication, first to review the experimental data collected in Kron's paper, particularly those referring to the relationship between the action It of the light-source and its actinic effect on the plate, and secondly, to devise a practical method by which photographic magnitudes of stars, strictly based on these experimental facts, may be obtained.

§ 1. *Discussion of Dr. Kron's Experimental Results.*—Without entering into the detail of the work, it may suffice; for the immediate purposes of astrophotometric investigations, to start from the series of curves shown numerically in Table 17 and graphically on plate ii. of Kron's paper. These curves represent the relation between the action It and the intensity I (or rather between the logarithms of these quantities), for equal actinic effects (Curven gleicher Schwaerzung). The general character of the curves demonstrates at sight the incompetence of the Schwarzschild law to represent the observed facts over the whole range of intensities. Accepting equation (2) as the analytical expression of this law, it follows:

$$\log (It) = (1-q) \log I + \text{const.}; \quad q = \text{const.} \quad . \quad . \quad (3)$$

Hence, plotting $\log (It)$ as ordinates against $\log I$ as abscissæ, the locus should be a straight line making an angle $\tan^{-1}(1-q)$ with the axis of abscissæ. The observed curve, on the other hand, is represented in fig. 1 (Plate 13), which shows, as an example, Kron's observations made with plates of the trade-mark "Agfa-Diapositiv."

Apparently the Schwarzschild law can be considered approximately valid only at the extreme ends of the curve, *i.e.* for very large or very small values of I, where the curve reaches asymptotically the condition of the straight line expressed by equation (3). For a certain intensity, I_0, the action It required to produce a desired actinic effect assumes a minimum value. Kron therefore defines this particular intensity as the *optimal* intensity of the plate. There is a particularly remarkable property of the It curves which will be utilised with advantage later on, viz. their symmetry with reference to the minimum point. It will also be noticed that in the vicinity of this point the law of reciprocity ($q = 1$) is fulfilled.

For practical purposes it is convenient to establish an empirical analytical relation between the action It and the intensity I.

The geometrical character of the curves suggested to Kron two alternative forms of empirical formulæ, viz. :—

$$\log It = \text{const.} + a\sqrt{(\log \mathrm{I})^2 + b},$$
$$\log It = \text{const.} + \log (a\mathrm{I}^{a} + b\mathrm{I}^{-a}).$$

From a physical point of view neither of these formulæ can at present claim preference ; their merits must be judged solely by their capability of representing the observations, and even in this respect their utilisation beyond the range of the latter can only be allowed with a certain reserve. According to Kron's calculations the former equation would seem to render somewhat better justice to the observations than the latter. But its very slight advantage in this respect is scarcely sufficient to warrant the rejection of the alternative formula, which would appear particularly useful in so far that it merges into the Schwarzschild equation (3) when extreme intensities are approached.

The figures comprised in Table I. demonstrate to what extent the second formula satisfies the observed facts. If we assume I_0, t_0 to represent the values of I, t at the minimum point and write

$$i = \mathrm{I}/\mathrm{I}_0 \; ; \; \tau = t/t_0,$$

the equation takes the form of that of a catenary symmetrical about the minimum point, which may be written in the form :

$$i\tau = \tfrac{1}{2}\left[i^{a} + i^{-a}\right] \quad . \quad . \quad . \quad . \quad (4)$$

The constants I_0, t_0, a can be determined from the observed data shown in the first and second column. By means of (4) the numerical values of $\log (It)$ may then be computed for the arguments log I of the first column ; they are shown in the fourth column, while the fifth column contains the differences (observed – computed). In addition two further rows of figures, headed $(m - m_0)$ and $\log \tau$, are exhibited, which will be useful later on. They are derived from the observed data of columns 1 and 2 in the following manner. According to fundamental photometric definition, the relation between magnitude m and intensity I is expressed by the formula

$$\log \mathrm{I} = -0{\cdot}40m + \text{const.} \quad . \quad . \quad . \quad (5)$$

If we assume the optimal intensity I_0 to be correlated with the magnitude m_0, we have

$$m - m_0 = -2{\cdot}5 \log i = -2{\cdot}5 \text{ (column 1} - \log \mathrm{I}_0).$$

Further,

$$\log \tau = \log t - \log t_0 = (\text{column 2} - \text{column 1} - \log t_0).$$

Kron's investigation comprises four different brands of plates : two "slow" plates (Agfa-Diapositiv and Ideal-Diapositiv) and two "fast" plates (Schleussner and Seed 27). The material shown in Table I. has been somewhat condensed by combining into means three consecutive groups in each of the four series published in Kron's Table 17.

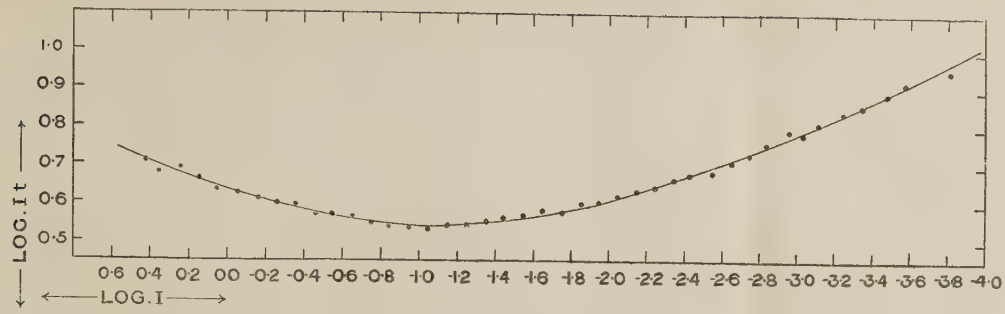
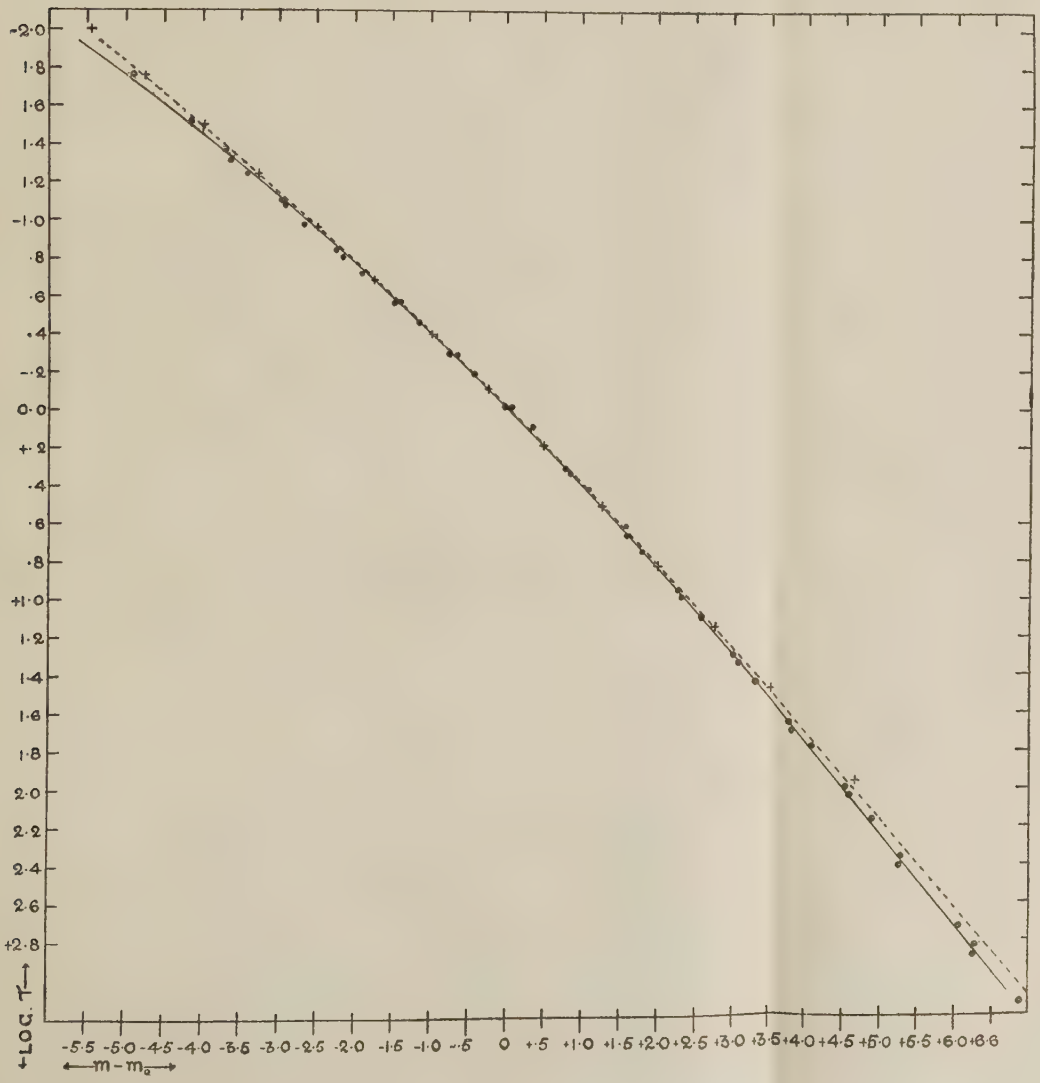

1·0
0·9
0·8
0·7
0·6
0·5
LOG. It
0·6 0·4 0·2 0·0 -0·2 -0·4 -0·6 -0·8 -1·0 -1·2 -1·4 -1·6 -1·8 -2·0 -2·2 -2·4 -2·6 -2·8 -3·0 -3·2 -3·4 -3·6 -3·8 -4·0
LOG. I

Fig. 1.


-2·0
1·8
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-·2
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2·2
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+LOG. r
-5·5 -5·0 -4·5 -4·0 -3·5 -3·0 -2·5 -2·0 -1·5 -1·0 -·5 0 +·5 +1·0 +1·5 +2·0 +2·5 +3·0 +3·5 +4·0 +4·5 +5·0 +5·5 +6·0 +6·6
m - m₀

Fig. 2.

TABLE I.

Agfa-Diapositiv. ($\alpha = 0{\cdot}26$; $\log I_0 = 8{\cdot}880$; $\log t_0 = 1{\cdot}670$.)

Observed. log I.	Observed. log *It.*	No. of Obs.	Compd. log *It.*	O−C.	$(m - m_0)$	log τ.
0·338	0·697	12	0·698	− ·001	−3·64	−1·311
0·050	0·643	30	0·649	− ·006	−2·92	−1·077
9·745	0·604	49	0·605	− ·001	−2·16	−0·811
9·449	0·573	82	0·575	− ·002	−1·42	−0·546
9·151	0·544	113	0·556	− ·012	−0·68	−0·277
8·848	0·538	163	0·550	− ·012	+0·08	+0·020
8·547	0·561	183	0·559	+ ·002	+0·83	+0·344
8·248	0·588	169	0·581	+ ·007	+1·58	+0·670
7·953	0·621	128	0·614	+ ·007	+2·32	+0·998
7·654	0·663	74	0·658	+ ·005	+3·07	+1·339
7·350	0·712	48	0·711	+ ·001	+3·82	+1·692
7·047	0·780	22	0·773	+ ·007	+4·58	+2·063
6·766	0·838	11	0·832	+ ·006	+5·28	+2·402
6·377	0·922	5	0·922	·000	+6·26	+2·875
	Ideal-Diapositiv.		($\alpha = 0{\cdot}19$; $\log I_0 = 8{\cdot}250$; $\log t_0 = 2{\cdot}363$.)			
0·431	0·786	6	0·787	− ·001	−5·45	−2·008
0·143	0·744	35	0·749	− ·005	−4·73	−1·762
9·844	0·705	47	0·711	− ·006	−3·98	−1·502
9·552	0·675	77	0·680	− ·005	−3·26	−1·240
9·250	0·651	112	0·653	− ·002	−2·50	−0·962
8·949	0·631	145	0·633	− ·002	−1·75	−0·681
8·647	0·615	162	0·619	− ·004	−0·99	−0·395
8·349	0·612	174	0·614	− ·002	−0·25	−0·100
8·053	0·624	137	0·615	+ ·009	+0·49	+0·208
7·748	0·637	93	0·628	+ ·009	+1·26	+0·526
7·452	0·652	56	0·639	+ ·013	+1·99	+0·837
7·143	0·662	28	0·662	·000	+2·77	+1·156
6·848	0·689	16	0·690	− ·001	+3·51	+1·478
6·385	0·708	10	0·744	− ·036	+4·66	+1·960
	Schleussner.		($\alpha = 0{\cdot}235$; $\log I_0 = 6{\cdot}880$; $\log t_0 = 2{\cdot}157$.)			
8·843	9·232	20	9·245	− ·013	−4·91	−1·768
8·547	9·186	26	9·193	− ·007	−4·17	−1·518
8·245	9·160	36	9·146	+ ·014	−3·41	−1·242
7·946	9·132	44	9·105	+ ·027	−2·67	−0·971
7·643	9·090	66	9·073	+ ·017	−1·91	−0·710
7·342	9·046	88	9·050	− ·004	−1·16	−0·453
7·052	9·025	113	9·039	− ·014	−0·43	−0·184

TABLE I.—Schleussner. ($\alpha = 0{\cdot}235$; $\log I_0 = 6{\cdot}880$; $\log t_0 = 2{\cdot}157$)—*contd.*

Observed. log I.	Observed. log It.	No. of Obs.	Compd. log It.	O − C.	$(m - m_0)$	$\log \tau$.
6·750	9·022	132	9·038	− ·016	+0·32	+0·115
6·452	9·038	116	9·048	− ·010	+1·07	+0·429
6·157	9·075	97	9·069	+ ·006	+1·81	+0·761
5·847	9·111	67	9·101	+ ·010	+2·58	+1·107
5·551	9·146	48	9·141	+ ·005	+3·32	+1·438
5·249	9·190	25	9·187	+ ·003	+4·08	+1·784
4·915	9·237	19	9·245	− ·008	+4·91	+2·165
4·362	9·346	9	9·354	− ·008	+6·29	+2·827
	Seed 27.	($\alpha = 0{\cdot}245$; $\log I_0 = 7{\cdot}360$; $\log t_0 = 1{\cdot}656$.)				
8·845	9·129	22	9·153	− ·024	−3·71	−1·372
8·541	9·102	28	9·105	− ·003	−2·95	−1·095
8·260	9·084	35	9·070	+ ·014	−2·25	−0·832
7·949	9·056	46	9·059	− ·003	−1·47	−0·549
7·658	9·027	65	9·022	+ ·005	−0·75	−0·287
7·346	8·999	106	9·017	− ·018	−0·03	−0·003
7·044	9·013	120	9·023	− ·010	+0·79	+0·313
6·752	9·038	117	9·040	− ·002	+1·57	+0·630
6·453	9·074	116	9·070	+ ·004	+2·27	+0·965
6·151	9·112	95	9·109	+ ·003	+3·02	+1·305
5·853	9·172	65	9·156	+ ·016	+3·77	+1·663
5·546	9·231	35	9·211	+ ·020	+4·53	+2·029
5·251	9·276	17	9·270	+ ·006	+5·27	+2·369
4·940	9·328	10	9·335	− ·007	+6·05	+2·732
4·613	9·403	5	9·405	− ·002	+6·87	+3·134

A systematic "run" in the differences O − C, though occasionally noticeable, is scarcely important enough to invalidate the adoption of equation (4), especially if we remember that an error of 0·020 in log I, which is exceeded only in three instances, is equivalent to only 0·05 magnitudes.

A striking feature of the analysis of Kron's figures is the consistency in the values of the exponent α for different plates. The analysis indicates that for

"Slow" plates:	Agfa-Diapositiv . .	$\alpha = 0{\cdot}26$
	Ideal-Diapositiv .	0·19
"Fast" plates:	Schleussner . . .	0·24
	Seed 27 . . .	0·25

If it were not for the doubtlessly real exception presented in the "Ideal-Diapositiv" brand, the conclusion might be warranted that α has the same value for all plates. As matters stand, however, the assumption of a constant α, though highly probable,

especially if our attention is confined to "fast" plates such as are used in astrophotometric work, is certainly open to criticism. Fortunately, however, a method can be pointed out in the course of this discussion by which the exponent α may be determined from certain evidence furnished by the particular plate from which star magnitudes are determined.

If the photometric relation (5) is introduced into (4), the latter equation may be written in the form

$$\tau = \tfrac{1}{2}\left[10^{0\cdot4(1+\alpha)(m-m_0)} + 10^{0\cdot4(1-\alpha)(m-m_0)}\right] \quad . \quad . \quad (6)$$

or

$$\tau = \tfrac{1}{2}10^{0\cdot4(1+\alpha)(m-m_0)}\left[1 + 10^{-0\cdot8\alpha(m-m_0)}\right].$$

For intensities considerably smaller than I_0, *i.e.* for values of m considerably greater than m_0, the bracket term on the right-hand side approaches unity, and hence

$$m = \text{const.} + \frac{2\cdot5}{1+\alpha}\log\tau, \quad . \quad . \quad . \quad (7)$$

whereas the reciprocity law would require

$$m = \text{const.} + 2\cdot5\log\tau.$$

Consequently the Schwarzschild exponent, p, is equal to

$$1/(1+\alpha).$$

The mean value of p for the four brands of plates is therefore 0·810, and the extreme values are 0·794 and 0·840. Astrophotometric determinations of this quantity, on the other hand, appear to centre round a mean value 0·80, in excellent agreement with Kron's experimental result.

Provided that in astrophotometric work we are dealing with intensities much smaller than the optimal intensity of the plate, the method advocated by Schwarzschild and adopted in my previous paper is free from objection. We shall see, however, that this condition is fulfilled only for comparatively faint stars. For bright stars the constancy of p in Schwarzschild's equation cannot be assumed, and the relation between m and t must therefore be derived from the more general equation (6).

The importance of equation (6) for astrophotographic work makes it desirable to represent it in a tabular form. Consequently, values of $\log\tau$ were computed for every tenth of the argument $(m-m_0)$ with three assumptions of α, viz. 0·30, 0·25, 0·20. The resulting figures are shown in Table II., which requires no further explanation. Further, to show how very satisfactorily equation (6) represents the observed data, I have plotted in fig. 2 (Plate 13) the values $(m-m_0)$ and $\log\tau$ of Table I. as abscissæ and ordinates, marking the observed points by circular dots in the case of Agfa-Diapositiv, Schleussner, and Seed 27, which are satisfied by practically the same value of α, and by small crosses for the exceptional plate Ideal-Diapositiv. The smooth drawn-out curve shows the related co-ordinates $(m-m_0)$ and $\log\tau$ taken from Table II. for $\alpha = 0\cdot25$, while the dotted curve refers to the same co-ordinates computed with the assumption $\alpha = 0\cdot20$.

TABLE II.

log τ.

Arg. $m-m_0$	$a=$ 0·30	0·25	0·20	Arg. $m-m_0$	$a=$ 0·30	0·25	0·20
−5·0	−1·674	−1·760	−1·837	−0·9	−0·347	−0·351	−0·354
−4·9	−1·645	−1·728	−1·803	−0·8	−0·309	−0·313	−0·315
−4·8	−1·615	−1·696	−1·769	−0·7	−0·272	−0·274	−0·276
−4·7	−1·585	−1·664	−1·734	−0·6	−0·234	−0·236	−0·237
−4·6	−1·556	−1·632	−1·700	−0·5	−0·196	−0·197	−0·198
−4·5	−1·526	−1·600	−1·665	−0·4	−0·157	−0·158	−0·159
−4·4	−1·496	−1·567	−1·631	−0·3	−0·118	−0·119	−0·119
−4·3	−1·466	−1·535	−1·596	−0·2	−0·079	−0·080	−0·080
−4·2	−1·436	−1·502	−1·561	−0·1	−0·040	−0·040	−0·040
−4·1	−1·406	−1·470	−1·526	0·0	0·000	0·000	0·000
−4·0	−1·376	−1·437	−1·491				
				+0·1	+0·040	+0·040	+0·040
−3·9	−1·346	−1·404	−1·456	+0·2	+0·081	+0·080	+0·080
−3·8	−1·315	−1·371	−1·421	+0·3	+0·122	+0·121	+0·121
−3·7	−1·284	−1·338	−1·386	+0·4	+0·163	+0·162	+0·161
−3·6	−1·253	−1·305	−1·351	+0·5	+0·204	+0·203	+0·202
−3·5	−1·222	−1·272	−1·315	+0·6	+0·246	+0·244	+0·243
−3·4	−1·191	−1·239	−1·280	+0·7	+0·288	+0·285	+0·284
−3·3	−1·160	−1·206	−1·244	+0·8	+0·331	+0·327	+0·325
−3·2	−1·129	−1·172	−1·209	+0·9	+0·374	+0·369	+0·366
−3·1	−1·097	−1·138	−1·173	+1·0	+0·416	+0·412	+0·407
−3·0	−1·065	−1·104	−1·137				
				+1·1	+0·459	+0·454	+0·449
−2·9	−1·033	−1·070	−1·101	+1·2	+0·503	+0·496	+0·491
−2·8	−1·001	−1·035	−1·065	+1·3	+0·547	+0·539	+0·532
−2·7	−0·969	−1·000	−1·028	+1·4	+0·592	+0·582	+0·574
−2·6	−0·936	−0·966	−0·992	+1·5	+0·636	+0·625	+0·616
−2·5	−0·903	−0·931	−0·955	+1·6	+0·681	+0·669	+0·659
−2·4	−0·871	−0·896	−0·919	+1·7	+0·726	+0·712	+0·701
−2·3	−0·838	−0·861	−0·882	+1·8	+0·772	+0·756	+0·743
−2·2	−0·804	−0·826	−0·845	+1·9	+0·817	+0·800	+0·786
−2·1	−0·771	−0·791	−0·808	+2·0	+0·863	+0·845	+0·829
−2·0	−0·737	−0·756	−0·771				
				+2·1	+0·909	+0·890	+0·872
−1·9	−0·703	−0·720	−0·734	+2·2	+0·956	+0·934	+0·915
−1·8	−0·668	−0·684	−0·697	+2·3	+1·002	+0·978	+0·958
−1·7	−0·634	−0·648	−0·659	+2·4	+1·049	+1·023	+1·001
−1·6	−0·599	−0·612	−0·621	+2·5	+1·096	+1·068	+1·045
−1·5	−0·564	−0·575	−0·584	+2·6	+1·144	+1·114	+1·088
−1·4	−0·528	−0·538	−0·546	+2·7	+1·191	+1·160	+1·132
−1·3	−0·493	−0·501	−0·508	+2·8	+1·239	+1·205	+1·175
−1·2	−0·457	−0·464	−0·469	+2·9	+1·287	+1·250	+1·219
−1·1	−0·421	−0·427	−0·431	+3·0	+1·335	+1·296	+1·263
−1·0	−0·384	−0·389	−0·393				

TABLE II.—log τ—*continued.*

Arg. $m-m_0$	$\alpha=$ 0·30	0·25	0·20
+3·0	+1·335	+1·296	+1·263
+3·1	+1·383	+1·342	+1·307
+3·2	+1·431	+1·388	+1·351
+3·3	+1·480	+1·434	+1·396
+3·4	+1·529	+1·481	+1·445
+3·5	+1·578	+1·528	+1·485
+3·6	+1·627	+1·575	+1·529
+3·7	+1·676	+1·622	+1·574
+3·8	+1·725	+1·669	+1·619
+3·9	+1·775	+1·716	+1·664
+4·0	+1·824	+1·763	+1·709
+4·1	+1·874	+1·810	+1·754
+4·2	+1·924	+1·858	+1·799
+4·3	+1·974	+1·906	+1 844
+4·4	+2·024	+1·953	+1·889
+4·5	+2·074	+2·000	+1·935
+4·6	+2·124	+2·048	+1·980
+4·7	+2·174	+2·096	+2·026
+4·8	+2·225	+2·144	+2·073
+4·9	+2·275	+2·192	+2·117
+5·0	+2·326	+2·240	+2·163
+5·1	+2·376	+2·288	+2·209
+5·2	+2·427	+2·336	+2·255
+5·3	+2·477	+2·385	+2·301
+5·4	+2·528	+2·434	+2·347
+5·5	+2·579	+2·482	+2·393
+5·6	+2·630	+2·530	+2·439
+5·7	+2·681	+2·579	+2·485
+5·8	+2·732	+2·628	+2·531
+5·9	+2·783	+2·677	+2·578
+6·0	+2·834	+2·726	+2·624
+6·1	+2·885	+2·774	+2·671
+6·2	+2·937	+2·832	+2·717
+6·3	+2·988	+2·872	+2·764
+6·4	+3·039	+2·921	+2·810
+6·5	+3·090	+2·970	+2·857
+6·6	+3·142	+3·019	+2·904
+6·7	+3·193	+3·068	+2·950
+6·8	+3·245	+3·117	+2·997
+6·9	+3·296	+3·166	+3·044
+7·0	+3·348	+3·216	+3·091

Arg. $m-m_0$	$\alpha=$ 0·30	0·25	0·20
+7·1	+3·399	+3·265	+3·138
+7·2	+3·451	+3·314	+3·185
+7·3	+3·502	+3·363	+3·232
+7·4	+3·554	+3·413	+3·279
+7·5	+3·605	+3·463	+3·326
+7·6	+3·657	+3·512	+3·373
+7·7	+3·709	+3·561	+3·420
+7·8	+3·761	+3·610	+3·467
+7·9	+3·812	+3·660	+3·514
+8·0	+3·864	+3·710	+3·561
+8·1	+3·916	+3·759	+3·608
+8·2	+3·968	+3·808	+3·656
+8·3	+4·019	+3·858	+3·703
+8·4	+4·071	+3·908	+3·750
+8·5	+4·123	+3·958	+3·798
+8·6	+4·175	+4·007	+3·845
+8·7	+4·226	+4·056	+3·892
+8·8	+4·278	+4·106	+3·940
+8·9	+4·330	+4·156	+3·987
+9·0	+4·382	+4·206	+3·034
+ 9·1	+4·434	+4·255	+4·082
+ 9·2	+4·486	+4·304	+4·129
+ 9·3	+4·537	+4·354	+4·177
+ 9·4	+4·589	+4·404	+4·224
+ 9·5	+4·641	+4·454	+4·272
+ 9·6	+4·693	+4·503	+4·319
+ 9·7	+4·745	+4·553	+4·367
+ 9·8	+4·797	+4·603	+4·415
+ 9·9	+4·849	+4·653	+4·462
+10·0	+4·901	+4·703	+4·510
+10·1	+4·953	+4·752	+4·557
+10·2	+5·005	+4·802	+4·605
+10·3	+5·056	+4·852	+4·653
+10·4	+5·108	+4·902	+4·700
+10·5	+5·160	+4·952	+4·748
+10·6	+5·212	+5·002	+4·796
+10·7	+5·264	+5·052	+4·843
+10·8	+5·316	+5·102	+4·891
+10·9	+5·368	+5·152	+4·939
+11·0	+5·420	+5·202	+4·986

§ 2. *Discussion of Properties of the Iso-Actinics.*—The curves shown in fig. 2 express the relationship between log i or $(m - m_0)$ on the one hand and log τ on the other for *equal actinic effects*. Curves of this kind may therefore be termed "iso-actinics." Kron's and Schwarzschild's experimental research and also the investigations of photographic magnitudes undertaken at this Observatory have established an important property of the iso-

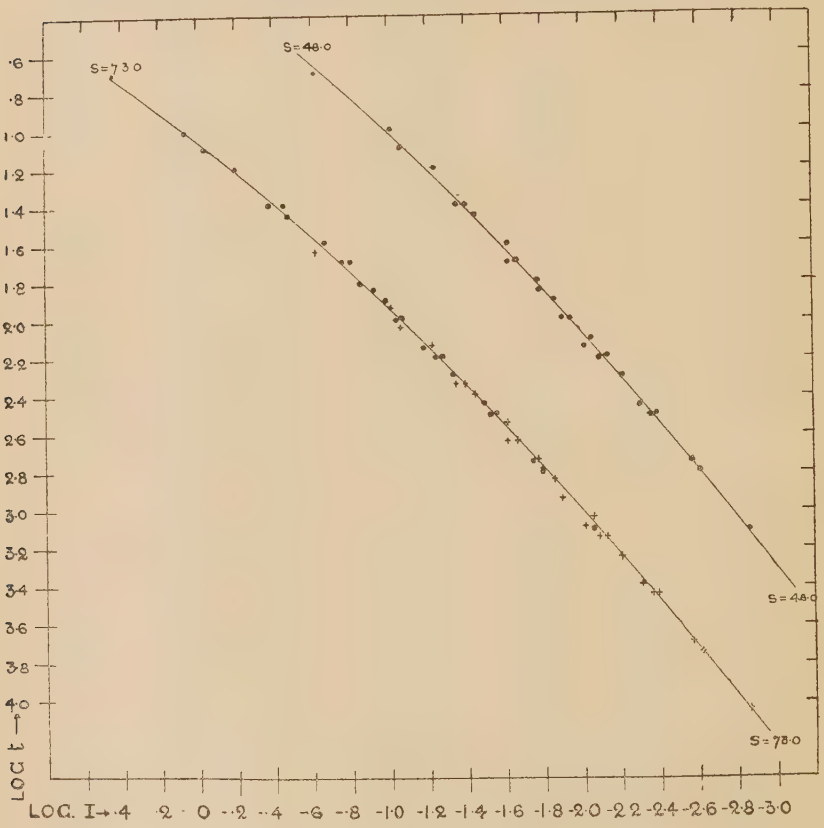


FIG. 3.

actinics which has proved of high value for the problem of determining fundamental photographic magnitudes. It appears that the iso-actinics belonging to different values of the actinic effects s are all similar in form and differ only in position by a constant shift parallel to the line of the log τ co-ordinates. As an illustration the iso-actinics for the extreme cases of s (viz. $s = 73{\cdot}0$ and $48{\cdot}0$) relating to Kron's 3rd series of the Agfa-Diapositiv plates are exhibited in fig. 3. The two curves are shown separately, the dots marking the observed data; but the points of the curve ($s = 48{\cdot}0$) have

again been plotted by taking as ordinates (log $t + 0{\cdot}94$) instead of log t, and are indicated in their altered positions by crosses. They fall well into line with the iso-actinic ($s = 73{\cdot}0$), thus showing that the two curves are identical in shape and differ only in their position along the log τ axis.

Analytically this property may be interpreted as follows. If t denote the time of exposure, I the intensity, and s the actinic effect, log t, m, s are related quantities, whose relationship may be supposed to be expressed in the form

$$\log t = \Phi(\log \mathrm{I}, s) \quad . \quad . \quad . \quad . \quad (8)$$

Now the result of Kron's experiment indicates that if s_1, s_2 denote two different values of s,

$$\Phi(\log \mathrm{I}, s_1) - \Phi(\log \mathrm{I}, s_2) \text{ is independent of } \log \mathrm{I};$$

or, if s_2 differs infinitesimally from s_1,

$$\frac{\partial \Phi}{\partial s} \text{ is independent of I,}$$

or

$$\frac{\partial^2 \Phi}{\partial s \partial \log \mathrm{I}} = 0.$$

Hence Φ is of the form $\phi(s) + \psi(\log \mathrm{I})$, where ϕ is a function of s only and ψ a function of log I only; therefore

$$\log t = \phi(s) + \psi(\log \mathrm{I}) \quad . \quad . \quad . \quad . \quad (9)$$

Equation (9) also expresses a property, similar to that found for the iso-actinics, applying to curves of equal intensity. Suppose that stars of different intensities are exposed for varying lengths of time and that curves be plotted for each star with s as abscissæ and log t as ordinates, then by virtue of (9) these curves are all identical in shape and differ only in position by a constant shift in the direction of the line of ordinates.

As an illustration of this property observations are selected from three plates containing exposures of the stars in the Pleiades, results of which—based on the Schwarzschild law—have already been published in my previous paper. On each plate nine exposures ranging from 3^s to 558^s were given. From the whole material of stars seven groups in order of brightness were then formed. For each of these groups the relation between the actinic effects (expressed in diameters of the star images) and log t are graphically shown in fig. 4.

The asserted property of these curves becomes apparent when the values of log t of different groups for the same diameters are examined. As an instance we find:—

Diameter.	log t. Group I.	Group II.	Δ log t.
0·90	1·75	2·67	0·92
0·85	1·57	2·50	0·93
0·80	1·39	2·31	0·92
0·75	1·19	2·12	0·93
0·70	1·00	1·93	0·93
0·65	0·79	1·72	0·93
0·60	0·58	1·50	0·92

The constancy of Δ log t throughout the whole range of diameters proves the correctness of the law expressed by equation (9).

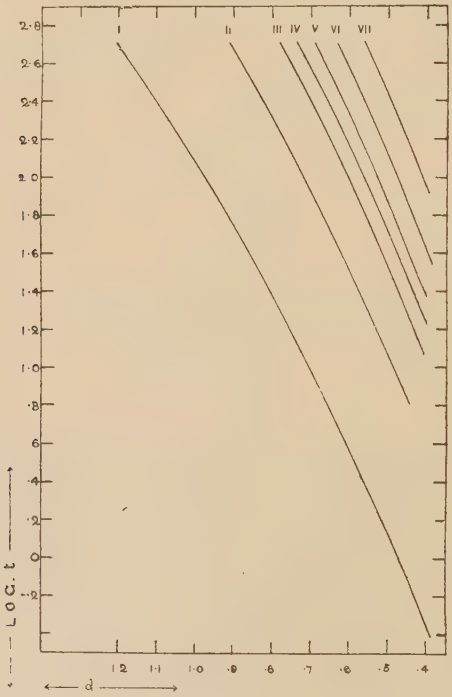


Fig. 4.

Owing to this property the curves of fig. 4 may be combined into one single curve, say that represented by Group I., on which subsequent calculations will be based.

§ 3. *Relation between the Intensities and Times of Exposure of Stars on any Plate and the Quantities i and τ.*—The material of a photographic plate, on which a number of stars have been exposed during varying lengths of time, may be examined in two

different directions. First, we may fix our attention on one star (or combined group of stars) of constant intensity I_c, and establish the relation between s (actinic effect) and $\log t$. This procedure is identical with that mentioned at the end of the last paragraph, and is graphically represented in fig. 4. Equation (9) may therefore be written

$$\log t = \phi(s) + \psi(\log I_c) \quad . \quad . \quad . \quad (10)$$

Or we may examine the actinic effects of objects of different intensity under the same conditions of exposure, *i.e.* $t = t_c = \text{const.}$ In this case equation (9) takes the form

$$\log t_c = \phi(s) + \psi(\log I) \quad . \quad . \quad . \quad (11)$$

Comparing equal actinic effects in both cases, we have the relation

$$\log t_c/t = \psi(\log I) - \psi(\log I_c) \quad . \quad . \quad . \quad (12)$$

But, in accordance with Kron's law, the intensity I exposed during time t produces the same actinic effect as the optimal intensity I_0 of the plate during time t_0, where $t/t_0 = \tau$.

Hence, according to (9)

$$\log \tau = \psi(\log I) - \psi(\log I_0) \quad . \quad . \quad . \quad (13)$$

and in virtue of (12)

$$\log t_c/t = \log \tau + a \quad . \quad . \quad . \quad . \quad (14)$$

where a is a constant.

Provided that a can be determined, and also that the plate supplies means for ascertaining the exact value of the exponent α, the value of $\log \tau$ can be found for each star by means of (14), and the corresponding value of $(m - m_0)$ is then supplied by Table II. In order to find the absolute magnitudes of the stars, the further knowledge of the magnitude of at least *one* star is necessary. On the whole, therefore, three data are still required, viz. the constants a, α, and m_0. We shall now proceed to show that at least two of these quantities, viz. a and α, are ascertainable from the internal evidence of the plate.

§ 4. *Determination of the Constant a.*—For the determination of this quantity it is required that the observations should be made with a wire grating in front of the object-glass.

The fundamental property of the wire grating consists in separating the light of a source into a series of components which, in the case of a star, are shown on the plate by a pattern containing a comparatively strong central disc flanked on either side by a row of fainter diffracted images. The principal central image will be henceforth denoted by the suffix p, the secondary diffracted images by the suffix s. Owing to colour dispersion the secondary images appear somewhat elongated, especially in the higher orders; with

the grating employed in the present investigation, however, the ellipticity of the lowest order image is so insignificant that the determination of its actinic effect by the diameter method is perfectly reliable. For this reason only the diffracted images of the lowest order will be utilised.

The difference in magnitude between the secondary and the principal image $(m_s - m_p)$ is known from the dimensions of the grating; it amounts in the present case to

$$m_s - m_p = 4^{m}\cdot 35.$$

Suppose now that the curve of equal intensity, represented by equation (10) and fig. 4, has been constructed from the principal images of a certain star by plotting the actinic effects (diameters) as abscissæ and $\log t$ as ordinates. Then, for the principal image of a star exposed during time t_c we obtain from (14)

$$\log t_c/t_p = \log \tau_p + a \quad . \quad . \quad . \quad (15)$$

where $\log t_p$ is found by reading off from the curve which represents equation (10) with the argument s_p.

Also we find for the secondary image

$$\log t_c/t_s = \log \tau_s + a \quad . \quad . \quad . \quad (16)$$

$\log t_s$ being obtained in like manner with the argument s_s.

Hence

$$\log t_p/t_s = \log \tau_s/\tau_p \quad . \quad . \quad . \quad (17)$$

and also

$$m_s - m_p = 4\cdot 35.$$

Let us now assume a series of values for $(m_p - m_0)$ such as are represented in the first column of Table III., and extract the corresponding values of $\log \tau_p$ from Table II., adopting the value of a determined by the method of the next paragraph. (Table III., as an example, is constructed on the assumption $a = 0\cdot 25$.) These values of $\log \tau_p$ are shown in the second column. Let us further construct in the same manner columns 3 and 4 for the secondary images, satisfying the condition $m_s - m_p = 4\cdot 35$. Hence the differences between the figures of columns 4 and 2 will give $\log \tau_s/\tau_p$ shown in the 5th column.

But the particular value of $\log \tau_s/\tau_p$ satisfying the conditions of the plate and the selected star is known from equation (17). Entering with this value into the 5th column of Table III., we find directly the corresponding $\log \tau_p$ and $\log \tau_s$. Finally, from equations (15) and (16) we get

$$\left.\begin{aligned} a &= \log t_c/t_p - \log \tau_p \\ &= \log t_c/t_s - \log \tau_s \end{aligned}\right\} \quad . \quad . \quad . \quad (18)$$

Table III.

$m_p - m_0$.	log τ_p.	$m_s - m_0$.	log τ_s.	log τ_s/τ_p.
−5·0	−1·760	− 0·65	−0·255	+1·505
−4·5	−1·600	− 0·15	−0·060	+1·540
−4·0	−1·437	+ 0·35	+0·142	+1·579
−3·5	−1·272	+ 0·85	+0·348	+1·620
−3·0	−1·104	+ 1·35	+0·560	+1·664
−2·5	−0·931	+ 1·85	+0·778	+1·709
−2·0	−0·756	+ 2·35	+1·000	+1·756
−1·5	−0·575	+ 2·85	+1·227	+1·802
−1·0	−0·389	+ 3·35	+1·457	+1·846
−0·5	−0·197	+ 3·85	+1·692	+1·889
0·0	0·000	+ 4·35	+1·929	+1·929
+0·5	+0·203	+ 4·85	+2·168	+1·965
+1·0	+0·412	+ 5·35	+2·409	+1·997
+1·5	+0·625	+ 5·85	+2·652	+2·027
+2·0	+0·845	+ 6·35	+2·897	+2·052
+2·5	+1·068	+ 6·85	+3·142	+2·074
+3·0	+1·296	+ 7·35	+3 388	+2·092
+3·5	+1·528	+ 7·85	+3·635	+2·107
+4·0	+1·763	+ 8·35	+3·883	+2·120
+4·5	+2·000	+ 8·85	+4·131	+2·131
+5·0	+2·240	+ 9·35	+4·379	+2·139
+5·5	+2·482	+ 9·85	+4·628	+2·146
+6·0	+2·726	+10·35	+4·877	+2·151

The accuracy in the determination of a naturally depends on the accuracy of log t_s/t_p, and also on the certainty with which this value can be located in Table III. Since the changes in log τ_s/τ_p are much more pronounced in the region of bright stars, we conclude that a must be determined by means of the *brightest* stars on the plate.

§ 5. *Determination of the Exponent* a.—A method exactly similar to that described in § 4 may be employed for the determination of a, only that in this case the *faintest* stars must be made use of. It was shown that for very small intensities Kron's law merges into Schwarzschild's law, which is expressed by equation (7), viz.:

$$m = \text{const.} + \frac{2{\cdot}5}{1+a} \log \tau \quad . \quad . \quad . \quad . \quad (19)$$

Suppose now that a faint star has been exposed during a time t_c, sufficiently great to produce a secondary image whose actinic effect (diameter) can be measured with accuracy. Again entering

with s_p and s_s into the curve representing equation (10), we determine log t_p and log t_s as before. According to (17)

$$\log t_p/t_s = \log \tau_s/\tau_p.$$

But from (7) or (19)

$$m_s - m_p = \frac{2{\cdot}5}{1+\alpha} \log \tau_s/\tau_p = \frac{2{\cdot}5}{1+\alpha} \log t_p/t_s = 4{\cdot}35.$$

Hence

$$\alpha = \frac{\log t_p - \log t_s}{1{\cdot}74} - 1 \quad . \quad . \quad . \quad (20)$$

Strictly speaking, the star employed in the determination of α should be of infinite magnitude. The value of α will always be too small when derived from stars of finite magnitude. We may, however, approximately represent the correct value by

$$\alpha = x\frac{\log t_p - \log t_s}{1{\cdot}74} - 1,$$

where $x > 1$. I have found that, using stars of the ninth magnitude, which may be supposed to be within the reach of this method, the factor x differs from unity only by 0·01.

§ 6. *Determination of the Constant* m_0.—According to definition, m_0 represents the magnitude of a star whose incident intensity is equivalent to the optimal intensity of the plate. Its value for any plate may be determined from the known magnitude of any one star on that plate. If, however, the optimal intensity could be assumed the same for all plates, it would be sufficient to determine m_0 once and for all from one or more standard magnitudes, such as, for instance, the Pleiades, and to adopt this value for all magnitude investigations. The problem of determining absolute photographic magnitudes independently of extraneous photometric data would then be solved.

An experimental examination of the changes in the optimal intensity I_0 from plate to plate is therefore of the highest importance.

Kron's investigations throw some valuable light on this point. First of all, the data given in Table I. show most emphatically that I_0 cannot be assumed constant for plates of different degrees of sensitiveness. In Table I. we found for

Agfa-Diapositiv . . .	log $I_0 = -1{\cdot}120$
Ideal-Diapositiv . . .	$-1{\cdot}750$
Schleussner . . .	$-3{\cdot}120$
Seed 27	$-2{\cdot}640$

When, however, attention is confined to plates of the same brand, we arrive at the important result that the changes of I_0 from plate to plate are insignificant, as long as the plates are submitted

to a perfectly uniform treatment in developing, but that I_0 changes sensibly when the treatment is varied.

As a typical illustration we may select Kron's observations of the Agfa-Diapositiv plates. In Table 15 of his paper he publishes the co-ordinates log I and log I*t* of the iso-actinics for seven series of observations, each series containing the mean results of a small number of plates against which the following details are marked in the ledgers:—

Series I.	Plates 259–263;	Developer: Rodinal	1 : 45;	6·5 mins.
II.	66–70	„	1 : 40;	5 „
III.	71–75	„	1 : 40;	5 „
IV.	99–91	„	1 : 40;	5 „
V.	112–115	„	1 : 40;	5 „
VI.	191–192	„	1 : 40;	1·5 „
VII.	189–190	„	1 : 40;	12·0 „

The iso-actinics constructed from Kron's figures are shown in fig. 5 with log I as abscissæ and log I*t* as ordinates.

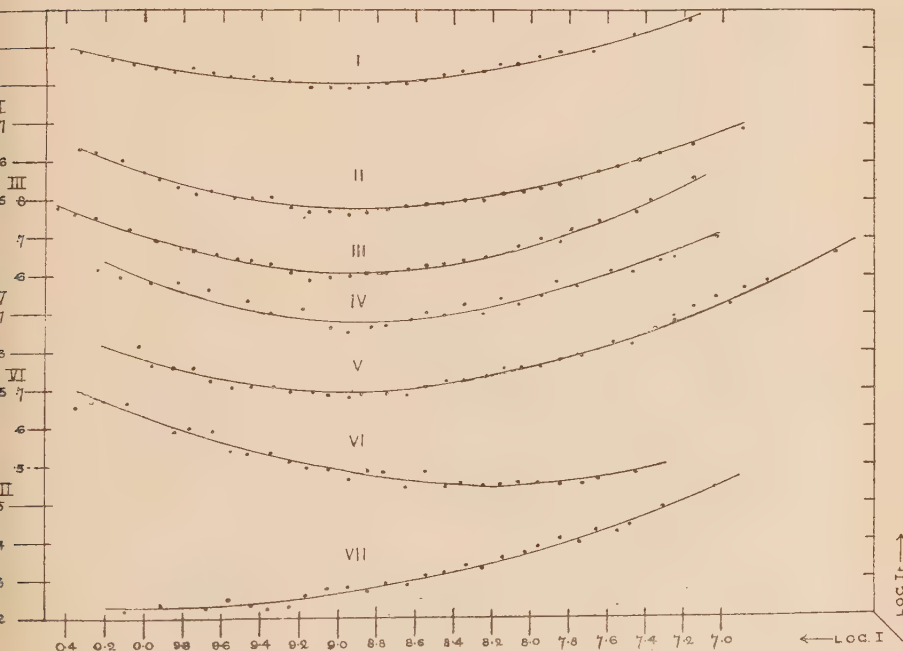


FIG. 5.

Comparing first those series in which the plates were developed under identical conditions as regards time and strength of the

developer (Series II.–V.), the constancy of I_0 appears to be remarkably pronounced. We find from the curves :—

Series II. . . .	$\log I_0 = -1\cdot20$
III. . . .	$-1\cdot10$
IV. . . .	$-1\cdot20$
V. . . .	$-1\cdot14$
Mean . . .	$-1\cdot16 \pm 0\cdot02$

On the other hand, a pronounced change with the duration of development is noticeable in Series VI. and VII., pointing to the conclusion that I_0 increases when the time of development is increased, and vice versa.

Provided, therefore, that precaution be taken to develop the plates under exactly the same conditions of time, concentration, and temperature of the developer, Kron's experiments appear to warrant the conclusion that m_0 may be assumed constant for plates of the same manufacture. We shall see later on that this result is strongly supported by star observations.

The question as to whether the constancy of m_0, even with the precautions referred to, can be relied on to such an extent that modern demands on the accuracy of magnitude determinations are satisfied, cannot be decided without further research. In experiments of the past the influence of the duration and mode of development has been seriously underrated, and for this reason we can scarcely claim to possess material fulfilling the necessary requirements. We also know next to nothing about the degree of homogeneity of the promiscuous brands of plates which have been used in astrophotographic work. The results of Kron certainly emphasise the necessity of the utmost care in the selection of plates for magnitude investigations and in their treatment in the developer.

Doubtless, therefore, of the three quantities α, a, and m_0 the last one is the most evasive, so far as changes from plate to plate are concerned. Not only is it affected by such changes of the optimal intensity as are indicated in fig. 5, but also by the varying conditions of definition and atmospheric absorption. Nevertheless, there are strong indications in favour of the view that difficulties in these various directions may eventually be overcome, and that *photographic* magnitudes, based on exact photometric principles, may be derived, independently of any data supplied by the results of *visual* photometry.

In connection with the changes of I_0 due to development, as shown in fig. 5, an explanation may now be given of a peculiar phenomenon mentioned in the previous paper. It was pointed out there that, if the Schwarzschild law be assumed and the exponent p be determined for each plate separately, the values of this quantity exhibit changes from plate to plate which correspond with similar fluctuations in the intensity of the actinic effect produced by the same star.

The adoption of the Schwarzschild law within a certain range of magnitudes implies that the Kron curve is considered a straight line inclined at an angle to the axis of abscissæ, whose tangent is derived from (4) by differentiation with respect to log i. Considering equation (3), we may write :—

$$(1-q)=\frac{p-1}{p}=\alpha\frac{i_m^{\alpha}-i_m^{-\alpha}}{i_m^{\alpha}+i_m^{-\alpha}} \quad . \quad . \quad . \quad (21)$$

where i_m represents the "mean" value of i for the interval considered. As an illustration of the changes which p undergoes when i_m is varied, I computed in Table IV. from (21) the p's for values of log i_m at regular intervals, assuming $\alpha = 0{\cdot}25$.

TABLE IV.

log i_m.	p.	log i_m.	p.
0·0	1·00	−1·6	0·85
−0·2	0·97	−1·8	0·84
−0·4	0·95	−2·0	0·83
−0·6	0·92	−2·2	0·82
−0·8	0·90	−2·4	0·82
−1·0	0·88	−2·6	0·82
−1·2	0·87	−2·8	0·81
−1·4	0·86	−3·0	0·81

Suppose now that the same stars have been observed on different plates ; then $I_m = \text{const.}$ for all the plates, and hence i_m varies inversely with I_0. But, according to fig. 5, I_0 increases with the duration of development. Hence, in agreement with Table IV., strongly developed plates show a smaller value of p than feebly developed plates. Since it may reasonably be assumed that the actinic effect of the same star is greater on the former than on the latter, we conclude that p derived from a plate on which the actinic effect d (diameter) or s (Schwaerzung) is abnormally large, is abnormally small, and vice versa. Hence the numerical relations mentioned in the previous paper, viz. :—

$p = 0{\cdot}821 - 0{\cdot}30\ (d - 1{\cdot}01)$ (Cape focal diameters)
$p = 0{\cdot}829 - 0{\cdot}028(s - 9{\cdot}2)$ (Schwarzschild's extra focal "blackening")

are qualitatively explained.

§ 7. *Summary of Method.*—In the practical application of the method the following scheme of observation and reduction may now be adopted :—

1. A series of exposures of varying lengths, denoted by t_c, t'_c, t''_c . . ., are taken with a wire grating in front of the object-glass, the dimensions of the grating being so selected that measurable actinic effects of the secondary images of stars equal to or fainter than ninth magnitude are obtainable.

2. The diameters of all the principal and secondary images are measured by micrometer.

3. The diameters of the principal images for various lengths of exposure are collected into groups, each group consisting of stars of approximately the same magnitude.

4. The corresponding values of d and $\log t$ for each group are plotted as abscissæ and ordinates and a smoothed curve is drawn through the observed points.

5. The curves of the individual groups are combined into one single curve by appropriate shifts in the direction of the $\log t$ co-ordinate. The resulting curve, which establishes the empirical relation between d and $\log t$ for constant magnitude and which extends over the complete range of diameters, will be referred to as the A-curve.

6. With the diameters d_p and d_s (belonging to the same time of exposure t_c) of the principal and secondary images of a number of the *faintest* possible stars, the A-curve is entered and the corresponding values of $\log t_p$ and $\log t_s$ are ascertained. The exponent α is then obtained from the relation

$$\alpha = 2{\cdot}5x\,\frac{\log t_p - \log t_s}{\Delta m} - 1 (x = 1{\cdot}01 \text{ approx.})$$

where Δm signifies the difference of magnitude between secondary and principal image derived from the dimensions of the grating.

7. With the diameters d_p and d_s (belonging to the same time of exposure t_c) of the principal and secondary images of some of the brightest stars the A-curve is entered and the corresponding values of $\log t_p$ and $\log t_s$ are ascertained. Then with the argument

$$\log t_p/t_s = \log \tau_s/\tau_p$$

a table similar to Table III., but based on the previously determined α and Δm, is entered and the corresponding values of $\log \tau_p$ and $\log \tau_s$ are extracted. Finally, for each star and exposure t_c the constant α is determined by the relation

$$\alpha = \log t_c/t_p \cdot \tau_p = \log t_c/t_s - \log \tau_s.$$

8. The quantities α and α being known, the calculation of the magnitude of every star can be proceeded with. These calculations may be based on the principal as well as the secondary images, but only the former will be here referred to.

In the case of every star, with the diameters d_p for all the observed times of exposure t_c, t'_c, t''_c . . . the A-curve is entered, the corresponding values $\log t_p$, $\log t'_p$ . . . are extracted and the logs of the ratios t_c/t_p, t'_c/t'_p . . . are computed. Apart from accidental errors, these logs should be identical; their mean is

therefore taken as the log t_c/t_p for the star in question. We then obtain

$$\log \tau_p = \log t_c/t_p - \alpha,$$

and entering with this quantity into Table II., the figures of which must be based on the derived value of α, we obtain $(m - m_0)$ for every star of the plate.

9. Finally, if the magnitude of one star be known from other sources, m_0 may be determined and the magnitudes m obtained by the addition of m_0.

The magnitudes thus derived are in strict accordance with the photometric law experimentally established by Kron for photographic plates in general.

The consistency in the values of α, which satisfy the individual series of each of the four brands of plates examined by Kron, leads to the conclusion that, at least as long as the same brand of plate is employed, this exponent may be safely assumed constant for all plates. Its evaluation might therefore be entrusted to special observations made in accordance with No. 6, and the derived value might then be accepted for observations on other plates of the same manufacture. The calculations would be considerably simplified owing to the fact that the same Tables II. and III. could be used throughout. From the experience gained in the course of these investigations, no hesitation is felt in recommending $\alpha = 0{\cdot}25$ as a value most likely to satisfy the conditions of such plates as are employed in this class of work. Nevertheless, it is important to show that the astronomical method discussed in § 5 offers the means for determining this quantity independently of the evidence adduced by laboratory experiments.

§ 8. *Determination of the Photographic Magnitudes of Stars in the Pleiades by the Preceding Method.*—As an example in illustration of the preceding method, I propose to reduce the results of three plates containing the stars of the Pleiades. Magnitudes of these stars, based on the Schwarzschild law, were already communicated in the paper referred to. It is therefore particularly interesting to see what improvement can be effected by the introduction of the more correct Kron law. Schwarzschild had already pointed out that his photographic results show unmistakable systematic differences when compared with the visual photometric results obtained at Potsdam. We shall find that these differences lose entirely their systematic character when Kron's law is adopted.

Unfortunately, the longest exposure on these plates is not sufficient to produce measurable secondary images of stars fainter than the seventh magnitude, owing to the considerable step in magnitudes between the principal and secondary images. We shall therefore accept $\alpha = 0{\cdot}25$ from Kron's investigation. Each plate contains nine exposures t_c, t'_c . . . ranging from $9^m{\cdot}3$ to 3^s, of which, however, in the case of the faintest stars, only the two longest show measurable diameters in the principal images.

As regards the A-curve, its construction has already been dis-

cussed in § 2, where it is graphically shown in fig. 4. It need only be mentioned that the curve of Group I., supplemented by the transposed curves of the other six groups, has been finally adopted.

For the evaluation of the constant a the seven brightest stars on the plate were used. First the values of $\log t_c/t_p$, $\log t'_c/t'_p$ . . . were determined. Their internal agreement may be shown in the case of the brightest star η (Alcyone):

Exposure 1 ;	$\log t_c/t_p = -0\cdot25$	Exposure 6 ;	$\log t_c/t_p = -0\cdot34$
2	$-0\cdot27$	7	$-0\cdot32$
3	$-0\cdot32$	8	$-0\cdot33$
4	$-0\cdot35$	9	$-0\cdot31$
5	$-0\cdot41$	Mean	$-0\cdot322$

Next for each star $\log t_p/t_s$ was found in accordance with § 7, No. 7. Table V. contains these quantities and the number of images on which they depend.

TABLE V.

Star.	$\log t_c/t_p$.	$\log t_p/t_s$. $=\log \tau_s/\tau_p$.	No. of Images.
η	$-0\cdot322$	2·006	7
b	$+0\cdot033$	2·020	6
f	$-0\cdot026$	2·028	5
c	$+0\cdot134$	1·968	5
d	$+0\cdot221$	2·020	4
e	$+0\cdot278$	1·977	4
h	$+0\cdot560$	1·980	4
Weighted means	$+0\cdot078$	2·001	

Entering with $\log \tau_s/\tau_p = 2\cdot001$ into Table III., the corresponding value $\log \tau_p = 0\cdot441$ is found.

Hence

$$a = \log t_c/t_p - \log \tau_p = -0\cdot363 \pm 0\cdot095.$$

A considerable uncertainty in the individual values of $\log \tau_s/\tau_p$ is of course not surprising. The p.e. of a may perhaps appear large, but we shall see presently that an error of this amount is thrown almost entirely into the constant m_0, and therefore is of no consequence. It must also be added that the plates were taken under extremely bad atmospheric conditions, and consequently show images much below the average quality.

After having computed the quantities $\log t_c/t_p$, $\log t'_c/t'_p$ . . . for all exposures and taken the means for each star, we find, by subtracting a from these means, the values of $\log \tau_p$, and entering into Table II. (for $\alpha = 0\cdot25$) the corresponding $(m - m_0)$. The figures representing these quantities are given for every star in Table VI.

TABLE VI.

Star (Bessel's Notation).	log τ_p.	$(m-m_0.)$	Star.	log τ_p.	$(m-m_0.)$	Star.	log τ_p.	$(m-m_0.)$
η	+0·041	+0·10	12	+1·837	+4·16	33	2·355	+5·24
b	+0·396	+0·96	24	+1·827	+4·13	9	2·425	+5·38
f	+0·337	+0·82	s	+1·790	+4·06	1	2·393	+5·32
c	+0·497	+1·20	19	+1·880	+4·25	30	2·458	+5·45
d	+0·584	+1·40	29	+1·963	+4·42	18	2·541	+5·63
e	+0·641	+1·54	4	+2·018	+4·54	27	2·596	+5·74
h	+0·923	+2·18	22	+1·996	+4·49	13	2·596	+5·74
g	+1·166	+2·72	10	+2·098	+4·70	21	2·750	+6·05
k	+1·363	+3·15	39	+2·153	+4·82	2	2·766	+6·08
			17	+2·051	+4·61	15	2·796	+6·15
34	+1·416	+3·26						
l	+1·535	+3·52	37	+2·165	+4·84	36	2·833	+6·22
p	+1·561	+3·57	31	+2·201	+4·92	8	2·716	+5·98
32	+1·709	+3·88	20	+2·277	+5·08	25	2·733	+6·02
38	+1·755	+3·98	23	+2·217	+4·95	11	3·085	+6·74
			7	+2·305	+5·14			

We may here examine how far the resulting magnitudes are affected by an error in a of the amount of the p.e. stated above. Let us therefore assume for a three values: $-0·458$, $-0·363$, and $-0·268$, and with the corresponding values of log τ_p let us take $(m-m_0)$ from Table II. It will be sufficient to do this for the three stars η, p, 11 which mark the beginning, middle, and end of the series. We find:—

		$a=$			II.–I.	II.–III.
		−0·458. I.	−0·363. II.	−0·268. III.		
η	$m-m_0=$	+0·34	+0·10	−0·14	−0·24	+0·24
p		+3·77	+3·57	+3·37	−0·20	+0·20
11		+6·93	+6·74	+6·54	−0·19	+0·20

The almost perfect constancy of the differences II.–I. and II.–III. justifies the above assertion, that errors in a of the assumed amount are almost entirely thrown into the constant m_0, but do not sensibly affect the scale.

§ 9. *Comparison of Photographic with Visual Magnitudes.*—The stars discussed in the preceding paragraph have been visually examined with great accuracy at Potsdam. In a comparison between visual and photographic magnitudes the correction for "colour" must be taken into account. As a measure for this correction we may conveniently use Hertzsprung's "effective wave-lengths," determined by means of the grating method, and we may be allowed to assume that the correction to magnitude, on account of colour, is proportional to the effective wave-length. The zero-point of this correction may be fixed at $\lambda = 4100$. Each

star of Table VI. thus leads to an equation of condition of the form

$$(m - m_0) - \text{Potsdam vis.} = a + b(\lambda - 4100) \quad . \quad (22)$$

from which the constants a and b are to be found by a least squares solution.

Instead of the individual stars we shall discuss the means of the nine groups into which Table VI. has been subdivided. For these groups the following data are available :—

TABLE VII.

Group.	$(m - m_0.)$ m	Potsdam Visual Mag. m	Effective Wave-length (Hertzsprung).
I.	+0·90	3·95	4101
II.	+2·40	5·48	4098
III.	+3·64	6·81	4118
IV.	+4·20	7·12	4129
V.	+4·63	7·54	4141
VI.	+4·99	7·96	4144
VII.	+5·40	8·40	4141
VIII.	+5·95	8·82	4145
IX.	+6·24	8·74	4264

A method of least squares solution of equations of the type (22) leads to the following values :—

$$a = -3{\cdot}09\,; \quad b = +0{\cdot}0036.$$

The Cape photographic system is thus represented by

$$(m - m_0) + 3{\cdot}09,$$

and the Potsdam photographic system by

$$\text{Potsdam visual} + 0{\cdot}0036(\lambda - 4100).$$

In Table VIII. the results are finally collected in such a form that direct comparisons may be made between the Potsdam photographic system on the one hand and the Cape system C_2 of the present paper, based on Kron's law, on the other. In addition, the Cape system C_1 of the previous paper, based on Schwarzschild's law, and also Schwarzschild's Ottakring observations S are stated and compared with Potsdam.

TABLE VIII.

Group.	P.	C_2.	C_1.	S.	$C_2 - P.$	$C_1 - P.$	$S - P.$
I.	3·95	3·99	3·99	4·07	+0·04	+0·04	+0·12
II.	5·47	5·49	5·41	5·41	+0·02	−0·06	−0·06
III.	6·87	6·73	6·65	6·65	−0·14	−0·22	−0·22
IV.	7·22	7·29	7·22	7·13	+0·07	0·00	−0·09
V.	7·69	7·72	7·66	7·61	+0·03	−0·03	−0·08
VI.	8·12	8·08	8·04	7·97	−0·04	−0·08	−0·15
VII.	8·55	8·49	8·47	8·42	−0·06	−0·08	−0·13
VIII.	8·99	9·04	9·05	8·95	+0·05	+0·06	−0·04
IX.	9·33	9·33	9·35	9·41	0·00	+0·02	+0·08

The differences $C_2 - P$, $C_1 - P$, and $S - P$ are graphically represented in fig. 6. It appears that the adoption of Kron's law leads to a highly satisfactory systematic agreement between magnitudes derived from photographic and visual methods. The assumption of the Schwarzschild law, on the other hand, introduces a slight but marked curvature, which must be ascribed to the fact that this law is not the correct expression for the photometric relations involved in the problem. The discrepancies between the Kron and Schwarzschild law become much more strongly accentuated when magnitudes outside those of Table VIII. are considered. Thus a star whose magnitude, according to the Kron law, is $0^m{\cdot}00$, would be $+0^m{\cdot}72$ accord-

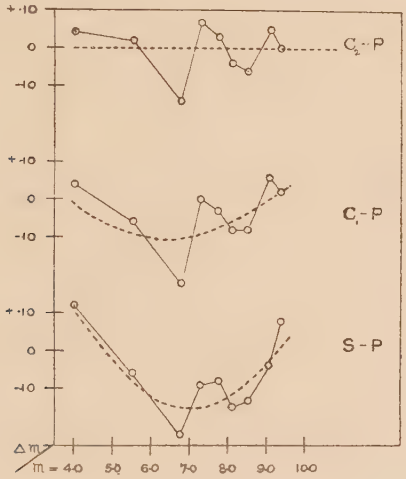


Fig. 6.

ing to Schwarzschild's law, and again a star of magnitude $14^m{\cdot}00$ according to the Kron law would be $14^m{\cdot}37$ according to the Schwarzschild law.

That the two laws should differ so much in the case of faint stars, although in that region the Kron law merges into the Schwarzschild law, is of course due to the fact that for the magnitudes of the column C_1 p was determined from the values of $\log t_s/t_p$ of the brightest stars, viz.:—

$$p = \frac{4{\cdot}35}{2{\cdot}5 \times 2{\cdot}001} = 0{\cdot}870,$$

whereas the Kron law adopted for column C_2 approaches for faint stars a Schwarzschild equation whose $p = \frac{1}{1+a} = 0{\cdot}800$. Hence there results a considerable difference in scale. The example

shows how unsafe it is to use the Schwarzschild law over a wide range of magnitudes.

§ 10. *Corrections to the Magnitudes of the South Polar Area.*— Since the magnitudes published in my previous paper are based on the Schwarzschild law, corrections will be required to reduce them to the Kron law. In this instance the principal and secondary images of the three brightest stars, $-89°, 47$; $-89°, 53$; $-89°, 34$, may be utilised for the determination of the constant a. We find for the mean of these stars, by combining the results of the six plates:—

$$\log \tau_s/\tau_p = 2{\cdot}108 \pm 0{\cdot}021 \; ; \; \log t_c/t_p = 0{\cdot}030.$$

But, according to Table III., the value of $\log \tau_p$ corresponding with $\log \tau_s/\tau_p$ is

$$\log \tau_p = 1{\cdot}530.$$

Hence

$$a = 0{\cdot}030 - 1{\cdot}530 = -1{\cdot}500.$$

For the determination of m_0 the photometric magnitude of $-89°, 47$ (σ Octantis) $= 5^m{\cdot}73$ has been adopted. The observations of this star show

$$\log t_c/t_p = -0{\cdot}290.$$

Hence

$$\log \tau_p = -0{\cdot}290 + 1{\cdot}500 = 1{\cdot}210,$$

and from Table II.

$$m - m_0 = +2{\cdot}81,$$
$$m_0 = 5{\cdot}73 - 2{\cdot}81 = 2^m{\cdot}92.$$

The agreement of m_0 with the value $3^m{\cdot}09$ found for the Pleiades is surprisingly good, and seems to corroborate the conclusion drawn from Kron's experiments that the optimal intensity is constant for plates of the same brand. (In both cases Ilford-Monarch plates were used.) The agreement must, however, be considered somewhat fortuitous, since according to Table III. the p.e. $\pm 0{\cdot}021$ of the quantity $\log \tau_s/\tau_p$ would correspond with a probable error in m_0 of approximately $\pm 0^m{\cdot}7$.

With the derived data we may now compute for a number of magnitudes the corresponding $(m - m_0)$ and $\log \tau_p$ according to Kron's law. These quantities are shown in the first three columns of Table IX.

On the other hand, the magnitudes in the previous paper were derived from the Schwarzschild equation:

$$m = 6{\cdot}32 + 2{\cdot}05 \log t_c/t_p = 6{\cdot}32 + 2{\cdot}05 \, (\log \tau_p + a),$$

or

$$m = 3{\cdot}25 + 2{\cdot}05 \log \tau_p \quad . \quad . \quad . \quad (23)$$

Substituting log τ_p into this equation the figures of the fourth column are found, which therefore represent the magnitudes according to the Schwarzschild law.

TABLE IX.

Kron Law.			Schwarzschild Law. (Equation (23.)	Correction
m.	$m - m_0$.	log τ_p. (Table II.)	m.	$(k - s)$.
m	m	m	m	m
5·00	2·08	0·881	5·06	−0·06
6·00	3·08	1·331	5·98	+0·02
7·00	4·08	1·801	6·94	+0·06
8·00	5·08	2·278	7·92	+0·08
9·00	6·08	2·764	8·92	+0·08
10·00	7·08	3·255	9·92	+0·08
11·00	8·08	3·749	10·93	+0·07
12·00	9·08	4·245	11·95	+0·05
13·00	10·08	4·742	12·97	+0·03

The corrections of the last column are to be applied to the magnitudes of the stars of the south polar area in my previous paper in order to reduce them to the photometric conditions expressed by Kron's law.

§ 11. *Influence of Variations in* α *and* m_0 *on Derived Magnitudes.* —Until confirmed by further experiments, the assumption $\alpha = 0{\cdot}25$ for all plates, on which the preceding calculations are based, must be accepted with reserve. Kron's investigation shows a noticeable departure from this value (viz. $\alpha = 0{\cdot}19$) in the case of the Ideal-Diapositiv plates, and although his "fast" plates are remarkably concordant, the presence of such an exception cannot be overlooked. It is, therefore, important to examine to what extent the derived magnitudes are changed, if for instance $\alpha = 0{\cdot}20$ be substituted for $\alpha = 0{\cdot}25$. The computations differ from those of § 8 only in so far that Tables II. and III. have to be used for the adopted value of α.

With regard to m_0, it was remarked that its determination depends on the accuracy with which log τ_s/τ_p of the bright stars can be ascertained. The p.e. of this quantity in the case of the Pleiades may be estimated at $\pm 0{\cdot}02$, so that discrepancies amounting to 0·06 are certainly possible. We shall, therefore, recompute the magnitudes of the nine groups on the assumption that log $\tau_s/\tau_p = 1{\cdot}94$ instead of 2·00, retaining $\alpha = 0{\cdot}25$.

The two solutions will be denoted by C_α and C_{m_0} respectively. They are shown in Table X., together with C_2 of Table VIII., the zero points being so chosen that the magnitudes of the first group agree in all the series.

TABLE X.

Group.	log t_c/t_p. (Observed.)	C_2 (Table VIII.).	C_a.	C_{m_0}.	$C_a - C_2$.	$C_{m_0} - C_2$.
I.	+0·010	3·99	3·99	3·99	0·00	0·00
II.	+0·660	5·49	5·44	5·55	−0·05	+0·06
III.	+1·232	6·73	6·68	6·83	−0·05	+0·10
IV.	+1·496	7·29	7·25	7·40	−0·04	+0·11
V.	+1·700	7·72	7·68	7·84	−0·04	+0·12
VI.	+1·870	8·08	8·05	8·20	−0·03	+0·12
VII.	+2·071	8·49	8·48	8·62	−0·01	+0·13
VIII.	+2·338	9·04	9·05	9·17	+0·01	+0·13
IX.	+2·480	9·33	9·35	9·47	+0·02	+0·14
X.	+5·294 (assumed)	15·00	15·28	15·16	+0·28	+0·16

To the nine groups, which embrace the actually observed stars, a tenth group is added showing the differences between the three assumptions for stars of the 15th magnitude, if in all cases the same quantity log $t_c/t_p = +5·294$ be assumed as the observed quantity. It appears that a star, whose magnitude according to assumption C_2 is 15·00, shows a magnitude 15·28 under C_a and 15·16 under C_{m0}. Differences of this amount are doubtless of significance in fundamental investigations of star magnitudes. They demonstrate the necessity of determining the quantities a and m_0 with a high degree of accuracy; particularly the former quantity, since the magnitude "scale" of the fainter stars depends largely on it. Whether a sufficient accuracy is practically attainable remains to be shown by further experiment. So far the results of observation are encouraging, and probably a stricter treatment of the plates in the process of developing may result in an even greater consistency of the results.

§ 12. *Changes of a and m_0 from Plate to Plate.*—It was pointed out in my previous paper (pp. 611–12), that changes from plate to plate are on the whole insignificant to such a degree that results derived from means of only two to three plates are highly concordant among themselves. A reliable test for this assertion is afforded by the quantities log t_c/t derived for the same stars from different plates. It was shown that these quantities differ from the log τ of Table II. by a constant. Hence, if the constants a and m_0 of Kron's equation have the same values for different plates, the quantities log t_c/t referring to the same stars on different plates should be brought into agreement by adding appropriate constants to their values on each plate. Thus, if the constants be so chosen that the quantities (log t_c/t + const.) agree for one particular star, they should, in the absence of changes in a and m_0, likewise agree for all the other stars. To what extent the plates agree in this respect is demonstrated by the following values of (log t_c/t + const.), which comprise series of three and four plates specified at the heads of their respective columns in Table XI. The observations refer to

groups of stars of the south polar area arranged in order of magnitude, the constants being so chosen that $(\log t_c/t + \text{const.}) = 0$ for the first group.

TABLE XI.

Group.	Mean Magnitude of Group. m	log t_c/t+const. Three Plates with Grating.	Three Plates with Grating.	Four Plates without Grating.
I.	6·8	0·00	0·00	0·00
II.	8·9	0·99	0·98	1·02
III.	10·0	1·55	1·54	1·58
IV.	10·7	1·90	1·88	1·89
V.	11·0	2·06	1·99	2·06
VI.	11·4	2·25	2·18	2·22
VII.	11·7	2·44	2·37	2·39
VIII.	12·0	2·57	2·52	2·56

The close agreement between the three series appears to justify the assumption that the constants a and m_0 of Kron's equation do not essentially differ from plate to plate. It was pointed out in my previous paper that the assumption of their constancy may be extended even to observations taken with different instruments (*l.c.*, pp. 611–12). On the whole, there is strong evidence in favour of the view that over a range of about 6 to 7 magnitudes the values $m_0 = 3^{m}{\cdot}0$ and $a = 0{\cdot}25$ may be adopted universally, at least under the optical conditions on which the discussed observations are based. It is probable, however, that m_0 may vary with the aperture of the telescope.

If the universality of m_0 and a should be confirmed by further investigations, the problem of photographic magnitudes would assume an extremely simple character. It would be sufficient, then, to take a series of exposures—no grating being required in this case—and to determine the A-curve, *i.e.* the relation between actinic effect and time, in accordance with the instructions of § 7. This curve will then enable us to determine the quantities $\log t_c/t$ for every star. Now, if the magnitude m_x of one particular star be known, we enter with argument $(m_x - 3{\cdot}0)$ into Table II. and note the corresponding value of $\log \tau_x$. Then

$$a = \log t_c/t_x - \log \tau_x,$$

and this quantity subtracted from the values of $\log t_c/t$ for every star will lead to the corresponding values of $\log \tau$, which in turn directly determine m by means of Table II.

Royal Observatory, Cape of Good Hope:
1914 *October.*

Note on the Number of Dark Stars. By F. A. Lindemann.

There would appear to be no direct method to observe or count the stars whose temperature is too low to emit a considerable fraction of the energy radiated in the form of visible light. This is not surprising in view of the proportionality of the energy radiated to the fourth power of the temperature and the lack of instruments whose sensitiveness to infra-red rays is anything like that of the eye, the photographic plate, or the photo-electric cell to light. Perhaps a somewhat inexact statistical calculation of the number of dark stars may therefore be of interest.

This method stands and falls with the hypothesis that new stars, so-called Novæ, are due to collisions of stars. It will therefore be necessary to examine this hypothesis somewhat in detail before entering upon the subject proper.

To enable one to gain some idea of the quantitative relations involved, certain assumptions must be made as to the typical new star. For the purpose in view these assumptions need only be quite rough, as they are to serve merely as a means of comparing the various hypotheses suggested to account for the phenomenon.

A Nova would probably escape notice unless it became fairly bright. It is assumed therefore that the new stars attain at least the 5th magnitude in general. For the same reason a large change in brightness may be postulated, say on an average 5 magnitudes. Further, the new star may be assumed to be fairly far away from the sun, as the number of stars at a given distance is roughly proportional to the square of the distance. The average distance may be taken as at least 100 parsecs, or roughly $3 \cdot 10^{20}$ cm. Therefore the phenomenon to be explained is that a star at about 100 parsecs distance * should increase from about the 10th to the 5th magnitude in a few days. The earth receives about $1{\cdot}4 \cdot 10^{-9} \frac{\text{erg}}{\text{cm.}^2 \text{sec.}}$ in the form of light from a star of the 10th magnitude. The exact value varies of course with the temperature of the star. This is still more the case with the total quantity of radiant energy received. In this paper complete radiation is assumed, as the correction for selective radiation is comparatively small except in the case of gases at low pressure. Now a 10th magnitude star at a distance of 100 parsecs at a temperature of 5000° must have the diameter of the sun. At 4000° its diameter would be three times, at 3000° about twenty-four times, the sun's diameter. To increase its luminosity by 5 magnitudes, *i.e.* a hundredfold, its temperature would have to be increased from 3000° to 7100°, from 4000° to 13,000°, from 5000° to 24,500°, from 6000° to 41,000°,

* The distance is really much greater than this, and an average distance of 2000 parsecs is inserted in the final calculation. The arguments in favour of the collision hypothesis would be much stronger if a larger distance were assumed. It seemed preferable, however, to show that even such a small distance as 100 parsecs leads to quite untenable consequences on any other theory.

and so forth. This sudden heating may be supposed to come either from within or to be caused by some external event. It can hardly be attributed to chemical change, as any exothermic chemical reaction reverses at high temperatures, besides not accounting for any such quantities of heat.

The only other possible internal cause which might be suggested as producing such an enormous amount of energy is some radioactive process. The most notable property of radioactive phenomena is the impossibility of influencing them by any extremes of pressure and temperature, so that calling them into account for a new star means supposing them to behave contrary to all our experience.

The external causes which might be invoked to explain the heat produced all reduce to a collision of the star with some mass and the consequent conversion of *vis viva* into heat.

The suggestion that the star might run into a nebula would seem to be untenable for various reasons; for instance, because the increased heat actually observed necessitates a nebula of an altogether impossible density. If the speed of the star were, for instance, $100\,\frac{\text{km.}}{\text{sec.}}$, the heat produced per cm.^{2} per second on entering a nebula of density σ would be $\frac{10^7\sigma \cdot 10^{14}}{2} = 5 \cdot 10^{20}\sigma$.

The star is supposed to be of the 5th magnitude, that is, assuming a distance of $3 \cdot 10^{20}$ cm. to send out $1{\cdot}4 \cdot 10^{-7} \cdot 4\pi \cdot 9{\cdot}10^{40} = 1{\cdot}58 \cdot 10^{35}$ ergs per second in the form of light. In the most favourable case, namely, at 6000°, this would imply a total radiation of $3{\cdot}26 \cdot 10^{25}\,\frac{\text{erg}}{\text{sec.}}$, namely, a surface of $4{\cdot}27 \cdot 10^{24}$ cm., or an emission of $6{\cdot}92 \cdot 10^{10}\,\frac{\text{erg}}{\text{cm.}^2\,\text{sec.}}$. Thus the density of the nebula would have to be at least $1{\cdot}4 \cdot 10^{-10}$. This would correspond to hydrogen at a pressure of $1{\cdot}55 \cdot 10^{-6}$ atm., or about $\frac{1}{1000}$ of a millimetre at ordinary temperature. That such a density is most unlikely has been emphasised by Mr. Ranyard, whilst the implied radius of the star, about nine times the sun's radius, does not exactly render the explanation more plausible.

Another reason against this hypothesis is that it would lead to an increasing or at any rate constant temperature while the star was passing through the nebula. As the brightness of new stars does not in general increase for more than a few days, say at most 10^6 sec., the nebula could only be a hundred million kilometres thick, which would not seem to harmonise with the apparent diameters.

The only remaining hypotheses are collision with a swarm of meteors or with a dark star. At first sight the first of these suggestions might seem to have much to recommend it. It would lead to an enormous increase of temperature at the surface, and the rapid diminution of brightness would be accounted for

by the fact that the inside of the star remains cold and that the heat produced by the meteoric impacts is therefore soon radiated. On the other hand, on this assumption once the surface had cooled down the star should resume its normal appearance, whereas nearly all new stars turn into planetary nebulæ.* Now the energy necessary to expand a mass m_0 of gravitating matter from radius r_0 to radius r_1 is $\frac{3}{5}\gamma m_0^2\left(\frac{1}{r_0}-\frac{1}{r_1}\right)$ where γ is the constant of gravitation. The radius r_1 of a planetary nebula is great in comparison to the radius of the star, so that the energy $\frac{3}{5}\gamma\frac{m_0^2}{r_0}$ is required. As the energy to be derived from the body or bodies of mass m_1 falling on the star of mass m_0 is $\gamma\frac{m_0 m_1}{r_0}$, it follows that $m_1=\frac{3}{5}m_0$, *i.e.* the mass of the swarm of meteors would have to be $\frac{3}{5}$ of the star's mass. As they would have to strike the star within a few days, they would have to be concentrated in a comparatively small volume and would therefore be unstable. This objection applies equally to the hypothesis of a collision with a nebula.

These difficulties do not appear in the case of a collision with a dark star, as this would naturally lead to the evaporation of the whole mass and consequent formation of a planetary nebula. The rapid cooling which might at first sight seem to be a stumbling-block would actually be expected on account of the enormous increase of potential energy due to the separation of the molecules during the formation of the nebula. In other words, the heat would be used up in the heat of vaporisation and in the ensuing adiabatic expansion.

If we may assume, therefore, that at any rate a definite percentage of Novæ are the result of collisions, the equating of the number of collisions to be expected on the ordinary kinetic theory of gases to the number of new stars observed will give the density of the stars in space. The number of collisions in unit time may be calculated in the usual way. For a first rough approximation the size of the stars may be taken as equal to the mean value. The correction applied by Sutherland to the original formula for the free path is absolutely essential in the case of stars, however, as the attraction is so large. Fortunately, the force function is known in this case in contradistinction to the kinetic theory. The mean free path

$$l=\frac{1}{\sqrt{2}N\pi 4\rho^2\left(1+\frac{2}{v^2}\int_{2\rho}^{\infty}F(r)dr\right)},$$

N being the number of stars in unit volume, ρ their radius, v their average relative velocity $\frac{4}{3}$ of the average speed on the

* Simon Newcomb, *The Stars*, p. 138.

kinetic theory, and $F(r)$ the force acting between a star and unit mass at a distance r. As

$$F(r) = \frac{\gamma m}{r^2} = \frac{6{\cdot}66 \,.\, 10^{-8} m}{r^2},$$

m being the star's mass $\frac{4\pi}{3}\rho^3\sigma$, σ being the density,

$$l = \frac{1}{4\sqrt{2}\pi N\rho^2\left(1 + \frac{2}{v^2} \,.\, \frac{6{\cdot}66 \,.\, 10^{-8} \,.\, 4\pi\rho^3\sigma}{2\rho \,.\, 3}\right)}.$$

As $2{\cdot}58 \,.\, 10^{-7}\rho^2$ is great in comparison to v^2, this reduces to

$$l = \frac{3v^2}{16\sqrt{2}\pi^2 \,.\, 6{\cdot}66 \,.\, 10^{-8}\rho^4\sigma N},$$

or for the average life τ to $\tau = \frac{4}{3} \,.\, \frac{2{\cdot}01 \,.\, 10^5 v}{\sigma\rho^4 N}$.

Now the average life is the number of stars in the space under observation divided by the number of Novæ observed, which are assumed to be due to collisions, therefore

$$\tau = \frac{\frac{4\pi}{3} r^3 N}{n},$$

r being the distance within which the Novæ are observed, or roughly $\sqrt[3]{2}$ times the average distance R of the Novæ from the earth. Thus

$$N^2 = \frac{4 \,.\, 2{\cdot}01 \,.\, 10^5 v \,.\, n}{8\pi R^3\rho^4\sigma},$$

or

$$N = 1{\cdot}79 \,.\, 10^2 \frac{\sqrt{v \,.\, n}}{\rho^2\sqrt{\sigma R^3}}.$$

This formula is in C.G.S. units. If v_1 is measured in kilometres, ρ_1 in units of the sun's radius, R_1 in parsecs, and if n_1 is defined as the number of Novæ per year and N_1 as the number of stars per cube of one parsec, it becomes

$$N = 1{\cdot}13 \,.\, 10^7 \frac{\sqrt{v_1 n_1}}{\rho_1^2\sqrt{\sigma R_1^3}}.$$

Putting $v_1 = 26{\cdot}5 \,.\, \frac{4}{3}$, $n_1 = \frac{1}{10}$, $\rho_1 = 1$, $\sigma_1 = 1$, $R_1 = 2000$, one finds $N_1 = 233$. Prof. Eddington * finds about one bright star in $17\frac{1}{2}$ parsec^3 or $5{\cdot}7 \,.\, 10^{-2}$ per parsec^3. The above value would therefore mean that there are about 4000 times as many dark stars as there are bright ones. In itself such a large figure

* A. S. Eddington, *Stellar Movements and the Structure of the Universe*, p. 44.

hardly seems inherently improbable. It would imply that the average life of a star, *i.e.* the period between two collisions, is of the order $1{\cdot}3 \cdot 10^{13}$ years, and though we can hardly suppose a star to remain incandescent for more than $.10^{8}$ years, there is obviously no reason why it should not survive indefinitely in a cold state. As it is impossible that the stars should fade out of existence as soon as they have become too cold for us to perceive them, whereas a phenomenon sufficiently violent to break them up into meteors or to return them to a nebulous state would hardly pass unnoticed, but would present all the characteristics of a new star, this line of argument only tends to confirm to the premises on which this paper is founded.

There is another well-known argument in favour of a large number of dark stars to be found in the high value of the average speed of the stars. Prof. Eddington has shown that the bright star density leads to a period of the order $3 . 10^{8}$ years for their oscillations in the system.* Even assuming the stars we measure to be those near the centre whose velocity is greatest, their average speed of 26·5 kilometres would lead to an average elongation of the order 5500 parsecs, or even correspond to 44,000 parsecs for stars of 200 kilometres velocity. This would seem too large. Assuming 4000 times as many dark stars as there are bright ones, one finds an average semi-major axis of 87 parsecs, or about 700 parsecs for exceptional high-speed stars, which would seem possible, though rather small. If one takes account of the smaller star density at great distances one arrives at similar results. It must be remembered, however, that the figures used in the calculation of the number of dark stars were only inserted by way of example, and that more correct values might lead to a smaller figure.

Sidmouth:
1915 *January* 3.

Observations of the Transit of Mercury, 1914 *November* 6–7, *made at the Radcliffe Observatory, Oxford.* By Dr. Arthur A. Rambaut, F.R.S., Radcliffe Observer. (Plate 14.)

The transit of Mercury on November 6–7 was observed at the Radcliffe Observatory under fairly favourable conditions. In the early morning a thick fog prevailed, but this began to clear away before 9 a.m., and as the time predicted for ingress approached the Sun shone in a cloudless sky.

During the latter half of the transit thin clouds formed over the Sun, becoming densest shortly after 1 p.m. But at no time were the clouds thick enough to obscure the planet completely, and the conditions gradually became more favourable again towards the end. Although at ingress the sky appeared clear, at that time

* *Loc. cit.*, p. 255.

the images were tremulous and the definition rather poor. At egress, on the other hand, although somewhat veiled by thin cloud, the images were steady and the definition good.

The telescopes employed were:—

(1) The 24-inch photographic equatorial and the 7-inch finder attached to that instrument.

(2) The 10-inch "Barclay" equatorial, and

(3) The $3\frac{1}{4}$-inch "Marlborough" portable equatorial.

The 24-inch equatorial and 7-inch finder were employed by Dr. Rambaut, assisted by Mr. J. G. Balk. The latter instrument was furnished with a diagonal reflecting solar eyepiece and wedge, and with this the times of the four contacts were observed, a power of 126 being employed throughout. During the passage of the planet across the Sun photographs of the disc were taken at intervals with the 24-inch refractor.

The 10-inch Barclay telescope, in the hands of Mr. W. H. Robinson, was used for measuring the form of the planet's disc, whilst the Marlborough instrument was employed by Mr. Barrett for observations of the contacts.

Observations of the Contacts.

The particulars of these observations are as follows, the initials A.A.R. denoting Dr. Rambaut and H.B. representing Mr. Barrett:—

INGRESS.

External Contact.

Aperture.	Power.	Observer.	G.M.T.	Observer's Remarks.
In.			h m s	
7	126	A.A.R.	21 58 18·1	First suspicion of a depression on limb. At $21^h\ 58^m\ 28^s{\cdot}1$ this had developed into a distinct and steady notch.
$3\frac{1}{4}$	54	H.B.	21 57 55·6	First contact.

Internal Contact.

7	126	A.A.R.	21 59 32·1	First meeting of white cusps round the planet's limb. At $22^h\ 0^m\ 9^s{\cdot}1$ planet quite clear of Sun's limb.
$3\frac{1}{4}$	54	H.B.	21 59 55·8	Apparent internal contact. At $22^h\ 0^m\ 2^s{\cdot}8$ just decidedly clear.

At ingress the air was very unsteady and images ill-defined. Sun's limb boiling. The image of the planet as seen in the 7-inch finder was drawn out into a pear, or peg-top, shape between first and second contact. When first clear of the Sun's limb it was composed of a black central nucleus surrounded by a circular penumbral fringe. The latter seemed to disappear as the definition improved later [A.A.R.].

No spot was noticed on the planet.

EGRESS.

Internal Contact.

Aperture.	Power.	Observer.	G.M.T.	Observer's Remarks.
In.			h m s	
7	126	A.A.R.	2 6 43·1	First separation of the cusps.
7	126	A.A.R.	2 6 53·6	Last trace of light between the cusps.

Internal contact must lie between these limits.

External Contact.

7	126	A.A.R.	2 8 13·1	First suspicion of disappearance, possibly 2 or 3 secs. early. $2^h\ 8^m\ 23^s{\cdot}1$ is noted as being "certainly too late."

The definition at egress was decidedly better than at ingress. No black drop, ligament, nor deformation of the planet's image could be detected as it passed smoothly and steadily off the Sun's disc.

The times of the four contacts as computed (for Oxford) from the *Nautical Almanac* are :—

	G.M.T. h m s
External contact at Ingress . .	Nov. 6, 21 58 7
Internal ,, ,, . . .	,, 6, 22 0 21
,, ,, Egress . . .	,, 7, 2 6 44
External ,, ,, . . .	,, 7, 2 8 56

It will thus be seen that, with the exception of A.A.R.'s observation of the first contact and possibly the third contact, all the observed times are in advance of the predicted. The large difference between the observed and predicted times for the last contact is to some extent supported by the following note of Mr. Barrett's. That observer writes: "Sun behind trees: could not make a certain observation. Mercury seen at times; and although the contact was not actually observed, the impression given was that it occurred about half a minute before the tabular time."

Measures of the Diameter of the Planet during Transit.

During the transit across the Sun measures of the planet's diameter were made with the Barclay equatorial of 10 inches aperture, fitted with the Grubb bifilar embossing micrometer carrying a diagonal solar eyepiece of power 300. The micrometer measures were effected by Mr. W. H. Robinson, assisted by Mr. H. Barrett, who set the position circle and noted times, etc.

These measures of the diameter were made first in the parallel, Pos.-Ang. $90^\circ - 270^\circ$; then in Pos.-Ang. $120^\circ - 300^\circ$; and so on in steps of 30° around the planet. Seventy-nine pairs were thus obtained in 19 sets of (generally) 4 each.

At each setting of the position circle four observations were made in the following manner :—

(1) *Direct*, *i.e.* the wires were placed tangentially to the limbs, each wire being on the side of the field nearest the corresponding micrometer head.

(2) and (3) *Reversed*, *i.e.* the positions of the two wires were interchanged, the wires being then on the sides of the field opposite to the positions of the heads.

(4) *Direct*, *i.e.* the wires were brought back to their former position and the measure made as in (1).

The means are exhibited in Table I. following, each result given in the last column representing the mean of (1), (2), (3), and (4).

TABLE I.

Measures of the Diameter of Mercury.

Set.	G.M.T. 1914.	Position-Angle of Diameter.	Observed Diameter.
	h m	° – °	″
1	Nov. 6, 22 20	90 – 270	9·61 *
2	22 35	120 – 300	9·29
3	22 41	150 – 330	9·72
4	22 48	180 – 0	9·37
5	22 54	210 – 30	9·76
6	23 6	240 – 60	9·61
7	23 13	270 – 90	9·08
8	23 17	300 – 120	10·07
9	23 25	330 – 150	9·67
10	23 35	0 – 180	9·53
11	Nov. 7, 0 9	30 – 210	9·60
12	0 18	60 – 240	9·86
13	0 22	90 – 270	10·07
14	0 31	90 – 270	9·81
15	0 36	120 – 300	8·96
16	0 46	150 – 330	9·98
17	1 5	180 – 0	9·76
18	1 13	210 – 30	9·22 *
19	1 27	240 – 60	10·30 *

Collecting these results in groups according to the position-angle, we obtain the following table :—

TABLE II.

Position-Angle.	Measures of Diameter.				Mean.	Diff. from Mean.
° – °	″	″	″	″	″	″
90 – 270	9·61 *	9·08	10·07	9·81	9·65	−0·01
120 – 300	9·29	10·07	8·96	...	9·44	+0·20
150 – 330	9·72	9·67	9·98	...	9·79	−0·15
180 – 0	9·37	9·53	9·76	...	9·55	+0·09
210 – 30	9·76	9·60	9·22 *	...	9·59	+0·05
240 – 60	9·61	9·86	10·30 *	...	9·85	−0·21

* Half weight.

If we smooth the above by taking means of each successive three, we get the following values for the six diameters in the same order as above, viz :—

$9''{\cdot}65$, $9''{\cdot}63$, $9''{\cdot}59$, $9''{\cdot}64$, $9''{\cdot}66$, $9''{\cdot}70$.

Plotting these results, a variation of amplitude $0''{\cdot}12$ is indicated, if indeed the small differences between the results should not rather be attributed to unavoidable errors of observation. This slight variation would correspond to a compression of only about $\frac{1}{80}$, the greatest diameter lying in the direction $235^\circ - 55^\circ$ position-angle.

The probable error of an individual result in the last column of Table I. is $\pm 0''{\cdot}22$, and the mean observed diameter of the planet is $9''{\cdot}64 \pm 0''{\cdot}05$, the tabular value being $9''{\cdot}88$.

The planet's image was fairly good on the whole, with sharply defined, though unsteady, limbs, but occasionally, especially towards the beginning, bad definition caused a penumbral fringe to appear. Measures were made as far as possible at intervals of best definition. For a short time from $1^h\ 10^m$ to $1^h\ 15^m$ G.M.T. clouds passed over the Sun, and later the driving clock gave some trouble. For these reasons half weight only was assigned to the last two sets of observations. To the first set also half weight was given, as the *Direct* observation depends on only one measure.

Although not examining the image for details, Mr. Robinson states that no spot nor compression was noticed by him, and at the close of the measures Mr. Barrett, looking specially for the latter feature, was satisfied that the image was sensibly round.

Photographic Observations.

No attempt was made to photograph the planet at ingress and egress, but during its passage across the disc of the Sun, between the second and third contacts, a number of photographs were taken with the 24-inch equatorial of 22 feet 6 inches focal length. For this purpose a rapidly moving shutter with a narrow slit ($\frac{1}{16}$ inch wide) close to the focal plane was used and the aperture of the telescope was stopped down to 4 inches. Imperial Process backed plates of quarter-plate size were employed, the Sun's image in the principal focus being a little over 2·5 inches in diameter. A reproduction of one of these plates (D 52) is shown on Plate 14, in which the position of the planet is indicated by the arrows in the margin.

Each plate was exposed twice to the Sun's rays, the telescope being unclamped in R.A. for an interval of about 100 seconds between the exposures. Two overlapping images of the Sun were thus obtained on each plate in a manner somewhat similar to that adopted with the Thompson photo-heliograph at the Royal Observatory, Greenwich, for determining the zero of position-angles. On the Radcliffe Observatory plates, however, the second image of the Sun is placed centrally, and in consequence the centre of the first image is at a considerable distance from the centre of the

Mercury in Transit, 1914 November 6, $23^{h}\ 40^{m}\ 12^{s}{\cdot}1$ G.M.T.
Radcliffe Observatory, Oxford.

plate. In fact, the whole disc of the Sun in the earlier exposure is not contained within the limits of a quarter-plate negative. This is, however, of no consequence, as only that portion of the first image is required where it overlaps the second, for the purpose of determining the direction of the chord common to the two circumferences, which, with a small correction for the motion of the Sun in the interval between the exposures, determines the direction of the north and south line through the centre of the second solar image.

The plates were placed in the Grubb measuring machine with a photographic réseau superimposed so that this common chord should lie as nearly as possible in the direction of one of the axes of measurement. It was then turned through 90°, then through 180°, and finally through 270°, and measures made in all four orientations. The abscissæ corresponding to the extremities of the east-west diameter and the ordinates corresponding to those of the north-south diameter were measured, and both co-ordinates of the cusps (or extremities of the chord common to both solar images) as well as those of Mercury's centre were recorded. The position of the Sun's centre was taken to be the intersection of these two diameters, which is equivalent to neglecting terms of the second order of the standard co-ordinates in the corrections for refraction. Thus the rectangular co-ordinates of the two cusps and of Mercury's centre referred to the centre of the Sun were obtained.

In order to reduce the measured co-ordinates of Mercury to standard co-ordinates (so as to compare with the tabular places affected by aberration as interpolated from the *Nautical Almanac*) we have four corrections to apply.

These are the corrections for

(1) Refraction.
(2) Parallax.
(3) Error in setting the line of cusps parallel to one of the axes of measurement in the machine, and
(4) Inclination of the line of cusps to the true north-south line through the Sun's centre.

(1) For the refraction correction we have the well-known formulæ

$$d\xi = 1{\cdot}0156k\{(1+\mathrm{X}^2)\,.\,\xi + \mathrm{XY}\,.\,\eta\},$$
$$d\eta = 1{\cdot}0156k\{\mathrm{XY}\,.\,\xi + (1+\mathrm{Y}^2)\,.\,\eta\},$$

where ξ and η are the approximate standard co-ordinates of any point, X, Y those of the zenith projected on the plane of the plate, k the constant of refraction computed from Bessel's refraction tables, and 1·0156 the factor required to adapt it for photographic rays.

(2) The corrections for parallax are given by the formulæ

$$d\xi' = \{(p-\mathrm{P})\mathrm{X} - p\,.\,\xi\}\cos\zeta,$$
$$d\eta' = \{(p-\mathrm{P})\mathrm{Y} - p\,.\,\eta\}\cos\zeta,$$

in which P and p represent the equatorial horizontal parallaxes of the Sun and Mercury respectively for the time of conjunction in

R.A., viz. Nov. 6, $23^h\ 12^m$ G.M.T., and ζ denotes the zenith distance of the Sun's centre at the moment of exposure.

In these expressions the terms $p\xi \cos\zeta$ and $p\eta \cos\zeta$ are negligible, and the formulæ may be written

$$d\xi' = (p - \mathrm{P}) \,.\, \mathrm{X} \,.\, \cos\zeta,$$
$$d\eta' = (p - \mathrm{P}) \,.\, \mathrm{Y} \,.\, \cos\zeta.$$

(3) Having applied the corrections for refraction to the measured co-ordinates of the cusps (the effect of parallax on these is negligible), we find the corrected co-ordinates of the same points, namely, ξ_1, η_1 and ξ_2, η_2.

If therefore χ represents the tangent of the angle between the line of cusps and the direction of measurement (or the radian measure of the angle itself, since this angle is always very small), we have

$$\chi = \frac{\xi_2 - \xi_1}{\eta_2 - \eta_1}.$$

(4) If Δt is the time elapsed between the two exposures, $\Delta\tau$ the interval for which the telescope remains unclamped, if D_1 and D_2 denote the Sun's declination at the moments of first and second exposures, and if ψ be the angle between the line of cusps and the north-south line through the centre of the second image of the Sun, it is easy to see that

$$\tan\psi = \frac{\sin(\mathrm{D}_1 - \mathrm{D}_2) + 2\cos\mathrm{D}_1 \sin\mathrm{D}_2 \sin^2\tfrac{1}{2}\Delta\tau}{\cos\mathrm{D}_1 \sin\Delta\tau}.$$

Retaining only first-order terms in $\mathrm{D}_1 - \mathrm{D}_2$ and $\Delta\tau$ (of which $\Delta\tau$ is much the larger), this becomes

$$\psi = \frac{\mathrm{D}_1 - \mathrm{D}_2}{\Delta\tau} \sec\mathrm{D} + \tfrac{1}{2}\Delta\tau \sin\mathrm{D}.$$

In this formula D_1, D_2, $\Delta\tau$, and ψ are expressed in radians. If, however, we express $\mathrm{D}_1 - \mathrm{D}_2$ in seconds of arc and $\Delta\tau$ in seconds of mean time, we find

$$\psi = \frac{\mathrm{D}_1 - \mathrm{D}_2}{15\Delta\tau} \sec\mathrm{D} + \tfrac{15}{2}\Delta\tau \sin 1''.\sin\mathrm{D}.$$

But we find from the *Nautical Almanac* that the variation of D in 1 hour is $-44''{\cdot}70$, and therefore

$$\mathrm{D}_1 - \mathrm{D}_2 = +\frac{44{\cdot}70}{3600}.\,\Delta t = +0{\cdot}0124\,.\,\Delta t,$$

Δt being, like $\Delta\tau$, expressed in seconds of time. If, finally, we substitute the values of sec D and sin D corresponding to mid-transit, for which $\mathrm{D} = -16^\circ\ 7'$, we find

$$\psi = 0{\cdot}000860 \times \frac{dt}{\Delta\tau} - 0{\cdot}0000010 \times \Delta\tau.$$

It happens that in the case before us this correction is very small indeed, and might without sensible error have been entirely neglected. It will not, however, always be so insignificant. Its unimportance on this occasion is due to a coincidence—to the fact, namely, that the change in the Sun's declination in the interval Δt (the effect of which on ψ is represented by the first term above) happens very nearly to compensate for the curvature of the Sun's diurnal path, the effect of which is represented by the second term.

But although this correction might have been safely omitted, it has in all cases been combined with the previous correction χ, and applied as a final correction to the co-ordinates ξ and η. We thus obtain the complete orientation correction $(\chi+\psi)$ expressed in radians. The largest value of this quantity for the plates under consideration is 0·001363 ($=281''$) in the case of D 58, and for the other plates it is, in general, considerably less.

We thus deduce the final corrections to the rectangular co-ordinates as measured to convert them into standard co-ordinates referred to the Sun's centre, namely,

$$d\xi'' = -(\chi+\psi)\,.\,\eta,$$
$$d\eta'' = +(\chi+\psi)\,.\,\xi.$$

The co-ordinates of Mercury thus obtained are expressed in réseau intervals. The value of this unit expressed in arc was determined from measurements of stars photographed at the same focal adjustment of the telescope. In this way it was found that

$$1^{R} = 146''{\cdot}454.$$

This quantity might of course have been determined from the measures of the Sun's diameters. But the condition of the atmosphere at the time of the transit and the low altitude of the Sun seemed to render this mode of proceeding inadvisable. That the adopted value is very approximately correct is shown by the fact that the semi-diameter on November 7 as deduced with it comes out

$$16'\ 9''{\cdot}66,$$

a value which is intermediate between that deduced from Prof. Auwer's discussion of the Greenwich observations 1851 to 1883 (viz. $16'\ 10''{\cdot}16$, as given in the *Nautical Almanac*, 1914, p. 11, November) and that deduced from his discussion of heliometer measures (viz. $16'\ 8''{\cdot}59$, as given on p. 452 of the same work).

Twenty-four plates in all were exposed during the transit, in the interval between November 6, $22^{h}\ 17^{m}$, and November 7, $1^{h}\ 48^{m}$ G.M.T. In the case of one of these (D 46) a mistake was made and the exact time of exposure is uncertain. Another plate (D 61), although exposed correctly, was useless for measurement owing to clouds. Rejecting these, we have 22 plates suitable for measurement, from which the following results were obtained:—

TABLE III.

Plate No.	G.M.T. of Exposure.	$a_s - a_m$. Tabular.	$a_s - a_m$. Observed.	$a_s - a_m$. Tabular *minus* Observed.	$\delta_s - \delta_m$. Tabular.	$\delta_s - \delta_m$. Observed.	$\delta_s - \delta_m$. Tabular *minus* Observed.
D.	h m s	s	s	s	′ ″	′ ″	″
43	22 17 47·1	−19·81	−19·56	−0·25	+13 57·9	+13 58·7	−0·8
44	22 38·6	18·03	17·75	−0·28	13 45·5	13 45·5	0·0
45	31 49·6	14·65	14·50	−0·15	13 22·1	13 22·2	−0·1
47	46 58·6	9·07	9·10	+0·03	12 43·4	12 42·1	+1·3
48	22 53 53·6	6·53	6·49	−0·04	12 25·8	12 25·1	+0·7
49	23 5 44·6	−2·17	−2·14	−0·03	11 55·5	11 54·8	+0·7
50	17 10·6	+2·03	+2·15	−0·12	11 26·4	11 25·3	+1·1
51	31 35·6	7·34	7·48	−0·14	10 49·6	10 48·5	+1·1
52	40 12·1	10·50	10·57	−0·07	10 27·6	10 26·4	+1·2
53	49 32·1	13·93	13·87	+0·06	10 3·8	10 3·1	+0·7
54	23 56 23·1	16·45	16·64	−0·19	9 46·3	9 45·3	+1·0
55	0 17 52·1	24·35	24·51	−0·16	8 51·5	8 51·1	+0·4
56	26 13·6	27·42	27·55	−0·13	8 30·2	8 28·1	+2·1
57	37 37·1	31·61	31·78	−0·17	8 1·2	7 59·2	+2·0
58	44 7·6	34·00	34·07	−0·07	7 44·5	7 43·0	+1·5
59	0 55 5·6	38·03	38·19	−0·16	7 16·5	7 14·4	+2·1
60	1 0 18·6	39·95	40·06	−0·11	7 3·3	7 2·9	+0·4
62	14 27·1	45·15	45·23	−0·08	6 27·2	6 25·7	+1·5
63	24 32·6	48·86	48·93	−0·07	6 1·5	6 0·8	+0·7
64	30 5·1	50·89	50·99	−0·10	5 47·3	5 44·9	+2·4
65	40 30·1	54·72	54·79	−0·07	5 20·7	5 19·5	+1·2
66	1 47 3·6	+57·13	+57·22	−0·09	+5 4·0	+5 4·8	−0·8
			Mean	−0·109		Mean	+0·93

As the final result of these photographic observations we find

Corrections to the Tabular R.A. and Declination of Mercury, relative to the Sun.

$$\Delta\alpha = -0^{s}\cdot109 \pm 0^{s}\cdot012,$$

and

$$\Delta\delta = +0''\cdot93 \pm 0''\cdot12.$$

Radcliffe Observatory, Oxford:
1915, *January* 7.

ADDITIONAL NOTE.

It might here perhaps be well to direct attention to a slight defect in this method of observation, against which it would be advisable to take precautions on another similar occasion. It will be noticed that the preceding and following limbs in the second image of the Sun, which alone is used for measurement, are not

photographed under exactly similar conditions. The preceding limb falls on a part of the plate which has not previously been exposed to the Sun, whereas the following limb falls on a part where the first image has already been impressed. It might be supposed, therefore, that the difference of photographic irradiation would affect the readings of the X co-ordinates of the limbs and so the adopted position of the Sun's centre.

It is satisfactory to find that hardly any trace of a systematic effect of this kind can be detected in the measures of the 22 plates discussed above, the mean east-west semi-diameter agreeing with the mean north-south semi-diameter within 0″·07. It is found, however, that the following limb is more difficult to set upon in the measuring-machine than the preceding, the contrast between the former and the background adjoining being much less marked than in the case of the latter. This may add to the probable error of an observation.

Any danger of such a systematic effect might be avoided by taking a third overlapping exposure on the preceding side of the principal image. This would, it is true, get rid of a systematic error arising from this cause, but it would probably add to the accidental error of measurement, and is objectionable for other reasons.

A better method, and one not very difficult to apply, would be to fit an occulting shutter close inside the exposing shutter which carries the slit. This shutter might be made long and narrow so that in the first exposure, which is only required to give us the line of cusps, it would occult a zone of the Sun's image, including the east-west diameter. This shutter should be removed before the second exposure. The following and preceding limbs in the second exposure would then impress their images on virgin film and under exactly similar conditions.

Possibly a better plan still would be to make the occulting shutter cover the whole field with the exception of two small patches in which the images of the cusps would fall, whilst the rest of the plate would be affected by only the second exposure.

The Nebula H.V. 20 Ceti. By Mrs. Isaac Roberts.

(Plates 15, 16.)

The nebula H.V. 20 Ceti (*h* 57, G.C. 132, N.G.C. 247) was photographed at Starfield, Crowborough, in 1901, and is found, in the Isaac Roberts collection of celestial photographs, on the plate:

I.R. 3020, Dec. 9 1901, Exp. 1 h 30 m from 0 h 42 m to 2 h 12 m S.T. Crowborough,

and on the 5-inch camera plate 1350 obtained simultaneously with the 20-inch reflector plate 3020.

The nebula is described as follows in the Isaac Roberts Register of Celestial Photographs:

3020. H.V. 20 Ceti: sky clear, windy; stars round, not very numerous; spiral nebula, with stellar nucleus and numerous condensations; viewed at an acute angle; elongated 16′ *np* to *sf*.

Further Description.—The stellar nucleus, intensity 12 and about 8″ in diameter, is involved in fainter nebulosity, within which 6 or more very small knots are perceptible (kn_1 and kn_2 were measured).

The denser central region, measuring about 9′ *n* to *s* and 4′ *p* to *f*, elongated in position angle 172°, appears to be made up of very diffused arcs, the direction of which remains to be ascertained with a longer exposure in a more favourable latitude.

On the I.R. * plate 3020, the nebula H.V. 20 would appear as a left-hand spiral seen obliquely, of the Andromeda type.

One of the arcs, vaguely indicated by diffused nebulous condensations such as c_1, c_2, c_3, seems to be directed toward the group of condensations near the *p* margin of the nebula, on the following side of star 7; the arc then seems to run *sf*-ward to the stars 8 and 9. Another diffused arc is directed from condensation c_4 to c_5. South of the nucleus is a broad and pretty dense arc directed to c_8. A nebulous arc directed *np* to *sf* is perceptible between the stars 9 and A, while much fainter nebulosity extends south of star A.

North of the nucleus, the denser central region extends unto a curve passing near the stars 1 and 5, and remaining south of star 2. Farther north is an area relatively void of nebulosity, or filled with extremely tenuous matter with star 2 involved, and bordered by a broad nebulous band that limits the nebula on its *nf*, *n*, and *np* sides.

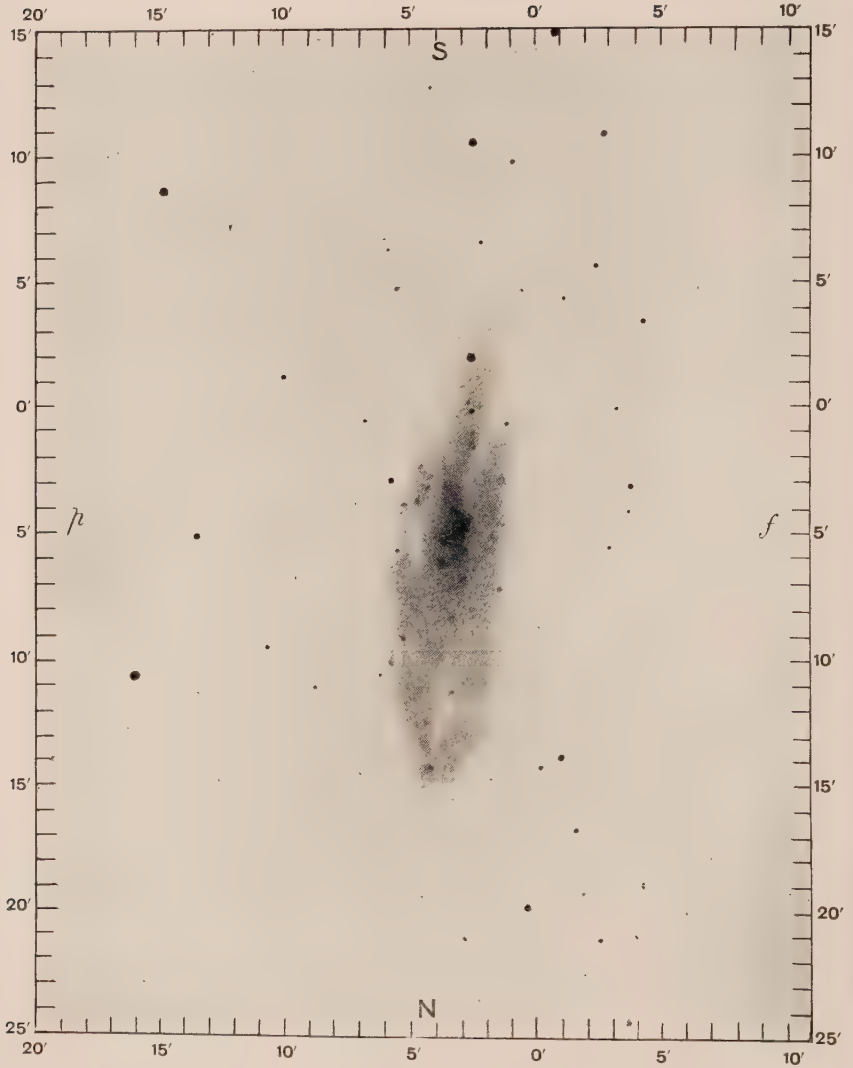
The marginal curve of the nebula on its *f* side is almost a straight line broken in the vicinity of the stars 1 and 10. Extremely faint nebulous knots such as kn_7, kn_8, kn_9 . . . are visible along the curve.

The stars encircling the nebula are more numerous in its southern than in its northern region. An area almost void of stars is formed by the stellar arc, 30, 31, 32, 33, 34, 35, 36; by the stars farther north, 37, 38, 39, 40, 41, 42, 43, 44, 45, situated beyond the northern limit of the drawing; by the stars 46, 47, situated beyond the *p* limit; and by the stars 48, 49, and 13 (see enlargement and measures).

The stars on the I.R. plate are in general well defined; the fainter ones are diffused.

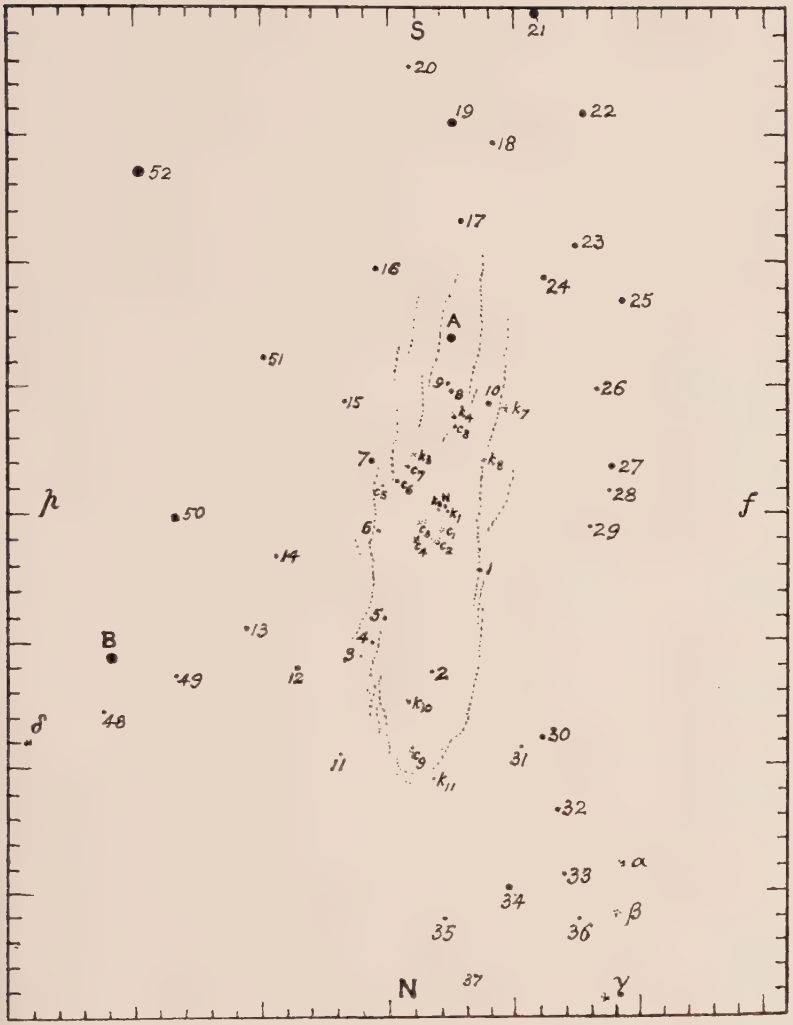
In the *nf* region of H.V. 20 are the three objects α, β, γ; in the *np* region the object δ of nebulous aspect. α and γ appear like small elongated nebulæ with dense nebulous nuclei. The objects β and δ are more diffused; they are perhaps spurious.

With its fainter outlines, the nebula H.V. 20 covers 19′ north



The Nebula H.V. 20 Ceti. I.R. ✳3020.

(Central portion of plate.)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k10
k11
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c2
c3
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c6
c7
c8
c9

Key Map.

to south and 5′ *f* to *p*. As stated above, it is elongated in position angle 172°, and by its general form recalls to the mind, on a very small scale, M 31 Andromedæ. The nebula is described by the Herschels as: "faint, extremely large, very much elongated 172°." It was not seen at Birr Castle, although looked for on one occasion.

Measures.—Two series of measures were made by me of the nebula H.V. 20 on the plate I.R. 3020, and the stars in its adjacent regions. The first series was made in May 1910; the second in the beginning of 1914. In the latter series all measures were made from the punctured centre of the plate: the standard stars are 8 in number. In the former series the measures were made from the bright star A; the standard stars measured were only 4 in number, and therefore the plate was re-measured in 1914. The constants of the plate I.R. 3020 in its two orientations on the I.R. stellar measuring apparatus, in 1910 and in 1914, were determined, as on former occasions, by Mr Pourteau of the "Bureau des Calculs de l'Observatoire de Paris." The positions of the standard stars in the System II. were derived from the *Catalogue d'Alger*, whereas the positions adopted for the determination of the constants in the System I. were those derived from the measures of the same stars in the System II. The fainter stars 8, 9, 30, and 38, together with the brighter stars, A, B, C, and F, were taken as standard stars in the System I.

The following tables contain the measures of all the objects studied on the plate I.R. 3020, and the results derived therefrom, together with a brief description of the objects:—

Plate I.R. *3020. *Systems* II_0. *and* I_A.

Places for 1900·0 of the Standard Stars from which the Constants were derived.

bj.	B.D.		Right Ascension. II_0.	Right Ascension. I_A.	Declination. II_0.	Declination. I_A.	Mag.	Authority. II_0.	Authority. I_A.
	° ′		h m s	″	° ′ ″	″			
E	−21 98	WZ	0 38 49·02	...	−21 37 6″·1	...	8·7	Algiers.	...
D	−21 104	A	0 39 53·74	...	−20 52 59·2	...	8·5	,,	...
C	−21 106	A	0 40 39·50	39·45	−21 27 20·7	20·6	8·7	,,	D.R. II.
B	−21 107	...	0 41 14·08	14·00	−21 12 34·3	33·7	8·5	,,	M. Pourteau.
A	−21 111	WZ	0 42 13·15	13·21	−21 24 58·0	57·7	8·9	,,	D.R. II.
F	−21 119	A	0 44 9·60	9·68	−21 41 44·1	44·9	7·0	,,	,,
G	−22 138	B	0 44 21·51	...	−21 56 35·3	...	8·2	,,	...
H	−21 123	A	0 44 45·99	...	−21 40 25·0	...	8·5	,,	...
30	...	...	0 42 ...	27·28	−21 9 ...	7·9	...	...	D.R. II.
38	...	...	0 42 ...	39·41	−20 57 ...	20·3	...	...	,,
*9	...	...	0 42 ...	12·40	−21 23 ...	15·4	...	...	,,
*8	...	...	0 42 ...	12·87	−21 22 ...	51·8	...	...	,,

Plate I.R. *3020. *Systems II_0. and I_A.*—Continued.

Obj.	Measures of Rectangular Co-ordinates. II_0. x_0.	II_0. y_0.	I_A. x_A.	I_A. y_A.	Concluded Positions for 1900·0 of the Standard Stars in the Systems II_0. and I_A. Right Ascension. II_0.	I_A.	Declination. II_0.	I_A.	Diameters. II_0.		I_A.		Intensity. II_0.	I_A.
	′	′	′	′	h m s	″	° ′	″	″	″	″	″		
E	−50·240	−13·173	...	...	0 38 49·10	...	−21 37	6·4	...	31	30	...	...	2	...
D	−34·555	+30·591	...	...	0 39 53·72	...	−20 52	58·4	...	31	30	...	...	2	...
C	−24·437	−3·831	−21·814	−2·300	0 40 39·45	39·47	−21 27	20·6	20·8	28	27	31	29	2	...
B	−16·145	+10·752	−13·753	+12·402	0 41 14·00	13·99	−21 12	34·1	33·8	27	26	28	28	2	...
A	−2·583	−1·848	0·000	0·000	0 42 13·21	13·22	−21 24	57·7	57·7	23	23	25	26	3	1
F	+24·144	−19·132	+26·981	−16·890	0 44 9·68	9·68	−21 41	44·9	45·1	37	37	38	38	1, 2	...
G	+26·520	−33·952	...	...	0 44 21·33	...	−21 56	33·6	...	38	38	...	...	2	...
H	+32·627	−18·017	...	...	0 44 46·08	...	−21 40	26·3	...	27	26	...	...	3	...
*30	+0·994	+13·862	+3·333	+15·747	0 42 27·28	27·27	−21 9	7·9	8·0	20	18	20	21	4	...
*38	+4·052	+25·558	+6·221	+27·478	0 42 39·41	39·43	−20 57	20·3	20·3	16	14	17	18	6, 16	2
*9	−2·741	−0·146	−0·189	+1·708	0 42 12·40	12·38	−21 23	15·4	14·9	7	7	8	9	14, 15	9
*8	−2·623	+0·247	−0·075	+2·095	0 42 12·87	12·86	−21 22	51·8	51·5	10	10	13	13	6, 15	4

The Constants referred to the Punctured Centre of the Plate I.R. *3020 and the Concluded Position thereof.

System II_0.

$t_x = +0·00002$ $t_y = +0·00323$

$i_x = -0·01921$ $i_y = -0·01945$

$\alpha_0 = 0^h\ 42^m\ 24^s·16$ $\delta_0 = -21°\ 23'\ 3''·5$

The Constants referred to Star A and the Concluded Position thereof.

System I_A.

$t_x = +0·00038$ $t_y = +0·00414$

$i_x = -0·00375$ $i_y = -0·00474$

$\alpha_0 = 0^h\ 42^m\ 13^s·22$ $\delta_0 = -21°\ 24'\ 57''·7$

The Nebula H.V. 20 Ceti (in the Systems II_0. and I_A.).

Obj.	Measured Co-ordinates. II_0. x_0.	y_0.	I_A. x_A.	y_A.	Concluded Places for 1900·0. Right Ascension. II_0.	I_A.	Declination. II_0.	I_A.	Diameters. II_0.	I_A.	Intensity. II_0.	I_A.
					h m s	s	° ′ ″	″	″ ″	″ ″		
H.V.20	− 2′945	+ 4′674	− 0′469	+ 6′497	0 42 11·13	11·10	− 21 18 25·6	26·4	8 8	12 12	12, 15	15, 19
*3	− 6·226	+ 10·555	...	...	0 41 56·58	...	− 21 12 35·2	...	6 3	... ...	16, 17	...
*4	− 5·834	+ 9·953	− 3·423	+ 11·759	0 41 58·31	58·34	− 21 13 10·9	10·2	14 9	11 11	10, 15	9
*7	− 5·786	+ 2·939	− 3·263	+ 4·747	0 41 59·07	59·13	− 21 20 13·2	12·6	16 14	17 18	6	2
c_5	− 5·074	+ 3·879	...	...	0 42 2·05	...	− 21 19 15·7	..	16 17	... ...	17, 18	...
c_6	− 4·670	+ 3·830	...	...	0 42 3·79	...	− 21 19 18·4	...	14 10	... ...	17	...
kn_{10}	− 4·423	+ 12·427	...	...	0 42 4·16	...	− 21 10 40·6	...	7 7	... ...	16	...
c_7	− 4·276	+ 3·237	− 1·762	+ 5·051	0 42 5·53	5·57	− 21 19 53·7	53·9	13 10	14 13	15	...
c_9	− 4·225	+ 14·191	...	...	0 42 4·87	...	− 21 8 54·2	...	24 16	... ...	15	...
kn_3	− 4·179	+ 2·734	...	...	0 42 5·99	...	− 21 20 23·8	..	6 6	... ...	16, 17	...
c_4	− 3·922	+ 6·123	...	...	0 42 6·82	...	− 21 16 59·5	...	9 7	... ...	18	...
c_3	− 3·726	+ 5·443	− 1·406	+ 6·857	0 42 7·71	7·07	− 21 17 40·2	65·0	15 19	18 19	17	17
*2	− 3·408	+ 11·250	...	...	0 42 8·61	...	− 21 11 50·3	...	10 10	... ...	15, 17	...
kn_{11}	− 3·399	+ 15·477	...	...	0 42 8·31	...	− 21 7 35·8	...	15 15	... ...	18	...
c_2	− 3·229	+ 6·034	...	...	0 42 9·80	...	21 17 4·0	...	16 14	... ...	18	...
kn_6	− 3·200	− 4·282	...	...	0 42 10·76	...	− 21 27 25·0	...	9 9	... ...	18	...
kn_2	− 3·118	+ 4·947	− 0·615	+ 6·789	0 42 10·37	10·47	− 21 18 9·4	8·9	7 6	11 8	15	17
c_1	− 2·930	+ 5·688	...	...	0 42 11·11	...	− 21 17 24·6	...	10 9	... ...	17	...

The Nebula H.V. 20 *Ceti* (*in the Systems* II_0. *and* I_A.)—Continued.

Obj.	Measured Co-ordinates. II_0. x_0.	y_0.	I_A. x_A.	y_A.	Concluded Places for 1900·0. Right Ascension. II_0.	I_A.	Declination. II_0.	I_A.	Diameters. II_0.	I_A.	Intensity. II_0.	I_A.
	′	′	′	′	h m s	s	° ′	″ ″	″ ″	″ ″		
kn_1	−2·761	+ 4·929	−0·268	+ 6·750	0 42 11·90	11·96	−21 18	10·0 11·1	6 5	13 9	15	17
*9	−2·741	− 0·146	−0·189	+ 1·708	0 42 12·40	12·38	−21 23	15·4 14·9	7 7	8 9	14, 15	9
*8	−2·623	+ 0·247	−0·075	+ 2·095	0 42 12·87	12·86	−21 22	51·8 51·5	10 10	13 13	6, 15	4
c_8	−2·520	+ 1·585	+0·008	+ 3·438	0 42 13·20	13·20	−21 21	31·0 30·6	10 7	13 8	15	14
kn_4	−2·502	+ 1·396	...	...	0 42 13·30	...	−21 22	42·4 ...	10 7	... ...	17	...
kn_5	−2·203	− 2·780	...	...	0 42 14·92	...	−21 25	53·4 ...	9 5	... ...	18	...
*1	−1·396	+ 7·195	..	...	0 42 17·58	...	−21 15	52·0 ...	10 10	... ...	14	...
*1	−1·393	+ 7·181	+1·056	+ 9·086	0 42 17·59	17·61	−21 15	52·9 50·0	9 9	10 12	14, 17	13
kn_8	−1·348	+ 2·763	...	...	0 42 18·14	...	−21 20	18·7 ...	17 15	... ...	18	...
*10	−1·136	+ 0·716	+1·404	+ 2·608	0 42 19·22	19·21	−21 22	21·7 20·1	12 10	11 13	12, 16	8
kn_7	−0·346	+ 0·912	+2·196	+ 2·803	0 42 22·60	22·61	−21 22	9·0 8·2	6 6	6 6	17	18
kn_9	+0·161	+ 4·019	...	...	0 42 24·52	...	−21 19	1·4 ...	9 7	... ...	17, 18	...

Small Stars in the Region of H.V. 20 Ceti.

Obj.	Measured Co-ordinates. II$_0$. x_0.	y_0.	Concluded Positions for 1900·0. Right Ascension. II$_0$.	Declination. II$_0$.	Diameters. II$_0$.	Intensity. II$_0$.
	′	′	h m s	° ′ ″	″ ″	
*46	−26·229	+21·607	0 40 29·98 ...	−21 1 51·1 ...	10 9	13 ...
*45	−25·513	+30·003	0 40 32·46 ...	−20 53 25·1 ...	15 15	8, 14 ...
*44	−24·328	+31·503	0 40 37·43 ...	−20 51 53·5 ...	18 17	8, 12 ...
*47	−22·595	+13·724	0 40 46·11 ...	−21 9 42·1 ...	14 13	6, 15 ...
*43	−19·847	+30·689	0 40 56·67 ...	−20 52 37·9 ...	11 10	13, 17 ...
*48	−16·440	+12·596	0 41 12·60 ...	−21 10 43·5 ...	11 10	15, 16 ...
*52	−14·826	− 8·656	0 41 21·12 ...	−21 32 1·0 ...	20 20	6 ...
*42	−14·451	+30·242	0 41 19·80 ...	−20 52 59·2 ...	20 19	5 ...
*50	−13·523	+ 5·168	0 41 25·67 ...	−21 18 7·6 ...	16 16	6 ...
*49	−13·502	+11·326	0 41 25·30 ...	−21 11 56·8 ...	10 9	16 ...
*41	−12·049	+33·808	0 41 29·82 ...	−20 49 22·1 ...	14 11	7, 14 ...
*13	−10·746	+ 9·537	0 41 37·26 ...	−21 13 41·4 ...	11 11	8, 14 ...
*51	−10·094	− 1·135	0 41 40·89 ...	−21 24 23·2 ...	11 10	8, 16 ...
*34	− 9·591	+ 6·830	0 41 42·43 ...	−21 16 23·2 ...	8 6	16 ..
*12	− 8·776	+11·032	0 41 45·60 ...	−21 12 9·4 ...	9 8	14, 17 ...
*11	− 6·986	+14·501	0 41 53·00 ...	−21 8 38 6 ...	13 9	16, 17 ...
*15	− 6·825	+ 0·625	0 41 54·79 ...	−21 22 33·7 ...	10 9	9, 14, 16 ...

Small Stars in the Region of H.V. 20 *Ceti*—Continued.

Obj.	Measured Co-ordinates. II$_0$ x_0.	II$_0$ y_0.	I$_A$. x_A.	I$_A$. y_A.	Concluded Positions for 1900·0. Right Ascension. II$_0$.	I$_A$.	Declination. II$_0$.	I$_A$.	Diameters. II$_0$.		I$_A$.		Intensity. II$_0$.	I$_A$.
	′	′	′	′	h m s	″	° ′ ″	″	″	″	″	″		
np	6·375	+10·555	...	...	0 41 55·94	...	−21 12 35·4	...	7	7	...	...	...	...
*16	5·465	− 4·726	...	...	0 42 1·06	...	−21 27 54·2	...	7	7	...	...	9, 16	...
,,	5·458	− 4·734	...	...	0 42 1·09	...	−21 27 54·7	...	7	6	...	...	7, 16	..
* 6	5·456	+ 5·700	−2·993	+ 7·538	0 42 0·27	0·25	−21 17 26·6	24·4	12	10	13	13	8, 14	8, 11
* 5	5·281	+ 9·092	−2·846	+10·903	0 42 0·75	0·83	−21 14 2·3	1·6	12	10	15	13	15, 16	11, 13, 17
*40	5·278	+31·434	...	...	0 41 58·98	..	−20 51 37·5	...	20	20	...	...	4	...
*20	4·166	−12·822	...	...	0 42 7·30	...	−21 36 0·1	...	7	8	...	...	15	...
*35	2·950	+21·080	...	...	0 42 9·78	...	−21 1 58·0	...	9	9	...	...	14, 16	...
*19	2·525	−10·471	...	...	0 42 14·17	...	−21 33 36·7	...	16	14	...	..	6, 15	...
,,	2·530	−10·466	...	...	0 42 14·14	...	−21 33 36·4	...	15	14	...	...	6	...
*37	2·315	+23·602	...	...	0 42 12·30	...	−20 59 25·5	...	10	10	...	...	18	...
*17	2·101	− 6·636	...	...	0 42 15·67	...	−21 29 45·4	...	11	10	...	...	8, 15	...
*18	0·909	− 9·772	...	...	0 42 21·06	...	−21 32 52·8	...	11	11	...	...	14, 17	...
*34	0·274	+19·891	...	...	0 42 21·35	...	−21 3 6·5	...	13	13	..	...	7	...
*39	0·163	+28·447	...	...	0 42 21·12	...	−20 54 31·4	...	12	10	...	...	10, 15	..
*31	+ 0·135	+14·357	...	...	0 42 23·56	...	−21 8 39·1	...	11	9	...	...	7, 15	...
*21	+ 0·883	−14·928	...	...	0 42 29·19	...	−21 38 1·0	...	16	14	...	...	6, 15	...
*30	+ 0·994	+13·862	+3·333	+15·747	0 42 27·28	27·27	21 9 7·9	8·0	20	18	20	21	4	...

*24	+ 1·085	− 4·416	...	...	0 42	29·19	...	−21	27	28·0	...	12	11	...	...	8, 13	...
*32	+ 1·564	+16·717	...	...	0 42	29·49	...	−21	6	15·4	...	13	11	...	...	8, 16	...
*33	+ 1·920	+19·255	...	...	0 42	30·80	...	−21	3	42·2	...	11	9	...	...	16	...
*23	+ 2·502	− 5·677	...	...	0 42	35·38	...	−21	28	42·3	...	13	12	...	...	8, 14	...
*36	+ 2·577	+21·154	...	...	0 42	33·46	...	−21	1	47·2	...	10	10	...	...	6, 8, 14	...
*22	+ 2·910	−10·882	...	...	0 42	37·57	...	−21	33	55·2	...	14	14	...	...	8, 14	...
*29	+ 2·983	+ 5·620	...	...	0 42	36·50	...	−21	17	21·7	...	10	10	...	...	8, 15	...
*26	+ 3·302	+ 0·128	+5·854	+ 2·062	0 42	38·34	38·34	−21	22	51·9	51·6	9	8	9	10	9, 16	6, 10, 11
*28	+ 3·736	+ 4·079	...	...	0 42	39·87	...	−21	18	53·6	...	10	9	...	...	15, 16	...
*27	+ 3·784	+ 3·222	...	...	0 42	40·14	...	−21	19	45·1	...	14	14	...	...	6, 15	...
*38	+ 4·052	+25·558	+6·221	+27·478	0 42	39·41	39·43	−20	57	20·3	20·3	16	14	17	18	6, 16	2
*25	+ 4·262	− 3·383	...	...	0 42	42·75	...	−21	26	22·2	..	14	13	...	...	7, 14	...
*53	+20·458	+16·802	...	...	0 43	50·49	...	−21	5	47·0	...	23	23	..	...	4	...

Diffused Objects of Nebulous Aspect.

Obj	Measured Co-ordinates. $x_{0.}$	$y_{0.}$	$x_{A.}$	$y_{A.}$	Concluded Positions for 1900·0. Right Ascension.		Declination.		Diameters.				Intensity.	
	II0.	II0.	IA.	IA.	II0.	IA.	II0.	IA.	II0.		IA.		II0.	IA.
					h m s	s	° ′	″	″	″	″	″		
δ	−19′·286	+14′·013	−17′·020	+15′·672	0 40	60·29	59·94	−21 9	20·8	17·4	28	24	12	13	18	15, 19
γ	− 3·653	+24·426	+ 5·868	+26·357	0 42	37·80	37·94	−20 58	28·9	27·9	11	9	14	13	11, 15, 17	9, 15
β	+ 4·031	+20·978	+ 6·295	+22·957	0 42	39·71	39·83	−21 1	56·1	52·5	24	20	23	16	17, 18	18
α	+ 4·198	+18·988	+ 6·493	+20·979	0 42	40·59	40·72	−21 3	55·6	51·7	11	6	6	5	16	13, 16

Definition and Description of the Objects Measured.

N, nucleus of H.V. 20, involved in fainter nebulosity.
kn_1, kn_2, very small knots in the immediate vicinity of N.
kn_3, irregular, other knots *sp*, the densest point measured.
kn_4, south of c_8.
kn_5, extremely faint, two small knots in line, the northernmost measured.
kn_6, extremely faint, other knots in vicinity.
kn_7, extremely faint, very small.
kn_8, extremely faint, irregular, group of small knots, difficult.
kn_9, irregular, very faint, beyond *f* margin of nebula.
kn_{10}, kn_{11}, large, *e.e.*F.
c_1, large (24″, 22″), with two nuclei, the denser measured.
c_2, large, diffused.
c_3, large, diffused, two different knots measured in the Systems II. and I.
c_4, small, three knots in line, the densest measured.
c_5, very diffuse, large, mottled.
c_6, irregular, with double nucleus.
c_7, the densest, in a curve of nebulous knots.
c_8, the densest measured.
c_9, 3 nuclei, whole mass measured.
α, elongated nebula, measuring about 1′ *n* to *s* and 30″ *f* to *p*, with a double or elongated nucleus (11″, 6″).
β, extremely faint, diffused, perhaps spurious.
γ, small nebula, elongated *sp* to *nf*, measuring about 1′ and 24″, with pretty dense nucleus, diameter 11″, 9″.
δ, diffused streak, two nebulous patches; the largest (28″, 24″), with a faint nucleus (12″, 13″), is perhaps spurious. (Confirmation of these objects is desired.)

Maison de Rosa Bonheur, By Thomery:
1914 *December* 4.

Observations of Occultations of Stars by the Moon, and Partial Eclipse of the Sun, made at the Royal Observatory, Greenwich, in the year 1914.

(*Communicated by the Astronomer Royal.*)

Day. 1914.		Phenomenon.	Mag.	Telescope.	G.M.T. h m s	Observer.
Feb.	3	Disapp. ϵ Arietis (N star)	6·0	Great Equat.	5 1 3·19	W.
	3	,, ϵ Arietis (S star)	5·7	,,	5 1 5·69	W.
	3	,, ,,	,,	Astrographic	5 1 6·57	R.S.
	3	,, W.B. (2) II. 1308	7·9	,,	8 17 44·14	W.S.
	3	,, W.B. (2) II. 1347–8	8·5	,,	8 36 41·26	W.S.
Apr.	2	,, B.D. +27°·984	9·4	,,	7 51 21·09	W.S.
	2	,, B.D. +28°·1011	9·2	,,	7 55 6·17	W.S.
	2	,, B.D. +28°·1016	8·9	,,	8 15 29·82	W.S.

Day. 1914.		Phenomenon.	Mag.	Telescope.	G.M.T. h m s	Observer.
Apr.	2	Disapp. W.B. (2) V. 1930	8·5	Astrographic	8 18 57·65	W.S.
	2	,, B.D. +27°·988	9·3	,,	8 20 11·45	W.S.
	29	,, 136 Tauri	4·6	Sheepshanks Finder	8 29 32·60	H.A.
	29	,, ,,	,,	Sheepshanks	8 29 33·00	W.D.
	29	,, ,,	,,	Astrographic	8 29 33·33	H.B.
May	4	,, 49 Leonis	5·7	,,	12 11 53·97	H.B.
June	4	,, 83 Virginis	5·6	Great Equat.	11 37 32·17	B.
Dec.	1	,, 17 Tauri	3·8	Detached Tel. (Altaz.)	6 7 4·72	A.C.
	1	,, ,,	,,	Merz	6 7 4·11	C.D.
	1	,, ,,	,,	Sheepshanks	6 7 4·48	W.
	1	,, ,,	,,	Astrographic	6 7 4·34	W.S.
	1	,, ,,	,,	Detached Tel.	6 7 3·89	W.A.L.
	1	,, ,,	,,	Thompson Eq.	6 7 4·41	E.M.
	1	,, B.D. +23°·512	7·3	Astrographic	6 31 30·40	W.S.
	1	,, 19 Tauri	4·3	Great Equat.	6 33 10·23	H.J.
	1	,, ,,	,,	Merz	6 33 10·79	C.D.
	1	,, ,,	,,	Astrographic	6 33 9·43	W.S.
	1	,, ,,	,,	Detached Tel.	6 33 10·63	G.B.
	1	,, ,,	,,	Thompson Eq.	6 33 10·99	E.M.
	1	,, 20 Tauri	4·1	Great Equat.	6 38 29·47	H.J.
	1	,, ,,	,,	Corbett Tel.	6 38 29·57	J.J.
	1	,, ,,	,,	Astrographic	6 38 29·78	H.
	1	,, ,,	,,	Detached Tel. (Altaz.)	6 38 30·46	A.C.
	1	,, ,,	,,	Merz	6 38 30·12	C.D.
	1	,, ,,	,,	Sheepshanks	6 38 30·14	W.
	1	,, ,,	,,	Detached Tel.	6 38 29·77	W.A.L.
	1	,, ,,	,,	Thompson Eq.	6 38 29·92	E.M.
	1	,, B.D. +23°·523	7·4	Astrographic	6 57 2·29	W.S.
	1	,, 22 Tauri	6·5	Great Equat.	6 58 8·16	H.J.
	1	,, ,,	,,	Astrographic	6 58 9·11	W.S.
	1	,, 21 Tauri	5·8	Great Equat.	6 59 29·84	H.J.
	1	Reap. 16 Tauri	5·4	Sheepshanks	7 11 57·05	W.
	1	Disapp. B.D. +24°·562	7·0	Great Equat.	7 19 40·35	H.J.
	1	,, ,,	,,	Astrographic	7 19 40·64	W.S.
	1	Reap. 19 Tauri	4·3	Great Equat.	7 25 35·69	H.J.
	1	,, ,,	,,	Astrographic	7 25 33·95	W.S.
	1	Disapp. B.D. +24°·566	7·5	,,	7 36 17·21	W.S.
	1	Reap. 20 Tauri	4·1	,,	7 46 8·48	H.
	1	,, ,,	,,	Sheepshanks	7 46 8·97	W.
	1	,, 21 Tauri	5·8	Great Equat.	7 48 20·19	H.J.
	1	,, 22 Tauri	6·5	,,	7 55 35·99	H.J.
	1	Disapp. Bradley 523	6·6	,,	8 20 15·97	H.J.
	1	,, ,,	,,	Astrographic	8 20 15·18	W.S.
	21	,, e^2 Aquarii	5·4	Detached Tel. (Altaz.)	7 2 40·05	G.B.

Observations of the Partial Eclipse of the Sun.

Day. 1914.	Phenomenon.	Mag.	Telescope.	G.M.T. h m s	Observer.
Aug. 20	(*a*) First contact	...	Astrographic	22 59 5·79	W.S.
21	Last contact	...	Great Equat.	1 20 41·48	W.
21	,,	...	Thompson Eq.	1 20 43·58	P.M.
21	,,	...	Astrographic	1 20 43·75	W.S.
21	,,	...	Merz	1 20 44·08	W.W.B.
21	,,	...	Detached Tel.	1 20 36·80	G.B.

(*a*) Not considered a good observation. Probably late.

The telescopes and clocks used are as follows :—

Great Equatorial . .	28 inches.	Clock—Dent 2009 and J. B. Dent 5658 (chronometer).
Thompson . . .	26 ,,	Mudge and Dutton (chronometer).
Merz Refractor . .	$12\frac{3}{4}$,,	Mudge and Dutton (chronometer).
Guiding Telescope of Astrographic Equatorial	10 ,,	Clock—Dent 2014.
Sheepshanks Equatorial	$6\frac{3}{4}$,,	,, Earnshaw.
Corbett . . .	$6\frac{1}{2}$,,	,, J. B. Dent 5658 (chronometer).
Detached Telescope .	4 ,,	,, ,, ,,
Detached Telescope (Altaz.)	3 ,,	Time tapped on chronograph.
Sheepshanks Finder .	2 ,,	Clock—Earnshaw.

The initials H.J., J.J., H., A.C., B., C.D., W., P.M., W.S., W.W.B., H.A., W.D., W.A.L., G.B., E.M., R.S., H.B., are those of Mr. Jones, Mr. Jackson, Mr. Hollis, Mr. Crommelin, Mr. Bryant, Mr. Davidson, Mr. Witchell, Mr. Melotte, Mr. Stevens, Mr. Burkett, Mr. Acton, Mr. Davies, Mr. Lambert, Mr. Bartle, Mr. Martin, Mr. Sims, and Mr. Barton.

Assuming that the error of the Moon's tabular place was wholly along the Moon's orbit, and that the Sun's tabular place needed corrections of $+^{s}\cdot06$ in R.A., $+''\cdot33$ in N.P.D., the observations of second contact in the eclipse of August 21 indicated that the tabular place of the Moon in *Nautical Almanac* needed corrections of $+0^{s}\cdot99$ in R.A., $+3''\cdot61$ in N.P.D.

The Sun and Moon were also observed on the meridian with the Transit Circle ; after correcting the observed R.A. of the Moon for the fact that the west point of her disc was slightly outside the Sun's disc, the corrections to the tabular co-ordinates are: Sun, R.A. $+^{s}\cdot10$; Moon, R.A. $+^{s}\cdot99$, Moon, N.P.D. $+2''\cdot6$ (correcting the tabular semi-diameter by $-2''\cdot3$ for irradiation). The corrections to the N.A. co-ordinates of the Moon, according to the Delaunay Tables (*M.N.*, lxxiii. 6, p. 413), are R.A. $+^{s}\cdot74$, N.P.D. $+2''\cdot6$.

A discussion of the Moon's errors was made from the observations of the occultation of the Pleiades on December 1.

The following are the corrections to the *Nautical Almanac* co-ordinates of the Moon from the Disappearances and Reappearances.

	Correction to R.A. of N.A.	Correction to N.P.D. of N.A.
	s	″
Disappearances	+0·87	−3·4
Reappearances	+0·69	−3·4

The difference between the two results in R.A. probably arises from the Reappearances (which were at the Bright Limb) being systematically observed late.

The meridian observations with the Transit Circle give $+0^{s}\cdot93$, $-3''\cdot2$.

The corrections to the N.A. co-ordinates, according to the Delaunay Tables, are $+0^{s}\cdot62$, $-2''\cdot6$.

New Double Stars. By Rev. T. E. Espin, M.A.

The measures of the following new double stars were made in the earlier part of the year with the 17¼-in. reflector, and in the latter part with the 24-in. reflector.

No.	B.D.	R.A. 1900. h m	Decl. ° ′	P. °	D. ″	Mags.		Nights.	Date. 1914.	
	+44° 4550	0 0·0	+44 40	119·8	13·32	6·5	13·5	1	·990	AB
				235·1	20·37	C=	9·2	2	·987	AC
1294	+44° 53	12·2	44 41	229·2	0·99	9·5	9·6	2	·984	
1295		23·4	43 50	320·6	4·03	9·6	9·8	3	·995	
1296		31·3	44 56	343·4	2·40	10·0	13·9	3	·388	
1297	+46° 192	48·1	46 58	240·9	4·97	9·7	9·8	3	·029	
1298	+45° 231	49·8	45 38	25·1	1·99	9·5	9·7	3	·906	
1299	+46° 212	52·7	46 40	247·8	5·38	8·7	12·8	3	·600	AB
				244·9	13·32	C=	13·5	3	·600	AC
1300	+45° 284	1 3·8	46 3	38·2	1·73	9·6	10·2	3	·853	
1301	+45° 433	38·8	45 49	283·8	2·12	9·5	9·8	2	·897	
1302	+44° 366	40·5	45 12	32·7	5·08	8·6	13·5	3	·987	
1303	+45° 445	40·8	45 44	240·8	7·46	9·3	13·7	2	·991	
1304		40·9	45 46	278·1	2·40	9·8	12·1	2	·991	
1305	+45° 448	42·2	45 30	274·5	5·97	9·5	9·6	2	·980	
1306	+44° 476	2 18·0	44 13	272·9	9·22	9·0	9·2	2	·994	
1307	+45° 635	30·5	45 20	3·4	2·11	9·5	9·7	4	·085	
1308	+44° 555	33·9	45 10	273·3	6·60	8·7	9·5	2	·042	
1309	+44° 592	45·1	44 21	214·6	3·23	9·3	13·9	3	·953	
	κ Persei	3 2·7	44 59	333·9	21·76	4·0	13·5	2	·994	
1310	+44° 639?	6·4	44 16	117·9	2·19	10·0	13·0	1	·997	BC
				158·4	20·21	A=	9·5	1	·997	AB
1311	+44° 671	14·1	44 51	28·5	5·18	9·3	13·8	3	·953	AB
				301·5	11·71	C=	9·7	3	·953	AC
1312	+44° 762	32·1	44 25	99·7	2·08	9·6	9·7	2	·994	
1313	+44° 803	45·2	44 40	21·9	2·97	9·5	14·0	2	·990	
1314	+45° 839	47·4	45 50	282·3	6·45	9·2	12·0	2	·105	

No.	B.D.	R.A. 1900. h m	Decl.	P.	D.	Mags.		Nights.	Date. 1914.	
1315	+46° 838	3 58·5	+46 57	317·4	9″·14	8·5	14·0	4	·091	
1316	+44° 912	4 12·7	44 37	78·5	4·80	9·5	10·7	2	·994	
1317	+44° 989	27·9	44 47	115·9	4·85	9·5	14·0	1	·890	
1318	+47° 1042	40·2	47 33	147·1	3·05	9·2	10·3	2	·064	
1319	+44° 1021	40·5	45 0	20·4	7·48	9·0	10·5	2	·994	
1320	+44° 1027	41·7	44 57	206·0	2·63	9·1	10·0	1	·991	
1321	+46° 1060	5 44·9	46 47	351·3	5·65	7·5	9·3	3	·167	
1322	+46° 1084	55·0	46 38	260·5	5·65	8·5	12·0	3	·177	
1323	+46° 1104	6 2·3	46 39	96·0	5·55	9·4	11·2	2	·183	
1324	+45° 1375	53·4	45 35	129·4	5·07	9·4	10·7	2	·181	
1325		7 53·8	45 8	355·6	2·75	9·4	10·1	3	·181	
1326	+44° 3035	18 57·4	44 51	229·2	2·46	9·3	10·4	2	·748	
1327	+44° 3174	19 30·3	44 27	51·6	2·36	9·3	9·4	3	·840	
1328	+43° 3337	40·2	44 5	88·1	2·51	9·6	13·5	2	·832	
1329	+44° 3340	20 2·9	44 43	215·6	6·53	8·0	10·5	2	·743	
1330		20·4	44 23	310·1	3·75	9·4	11·2	2	·832	
1331	+44° 3456	20·6	44 37	76·8	1·30	9·3	9·5	2	·756	
1332	+43° 3704	41·4	44 3	326·1	6·68	9·3	10·8	2	·871	
1333	+43° 3851	21 9·9	44 8	109·8	1·39	9·5	9·6	6	·746	
1334	+44° 3784	17·5	44 23	303·7	5·10	8·8	13·7	2	·694	
1335	+44° 3820	24·1	44 49	303·4	6·45	8·4	12·0	2	·725	
1336	+43° 3948	26·9	44 10	340·3	3·66	9·5	14·0	3	·877	
1337	+43° 3955	28·0	43 55	263·4	6·18	9·0	10·0	2	·894	
1338	+43° 3959	28·4	44 11	41·5	7·25	9·5	10·0	2	·873	
1339	+43° 3963	29·0	44 10	132·8	4·81	9·5	14·5	3	·877	AB
				247·1	23·63	C=	10·2	2	·876	AC
1340	+43° 3966	29·9	44 11	223·3	5·63	9·5	14·0	3	·891	
				130·9	5·65	C=	11·7	4	·894	
1341	+43° 4018	38·7	44 7	294·6	3·88	9·5	14·0	3	·889	
1342	+44° 4076	22 10·0	45 0	185·3	3·73	9·1	13·0	2	·719	
1343	+44° 4090	13·5	44 33	258·4	1·33	9·4	9·6	4	·865	
1344	+44° 4099	16·2	44 45	182·0	4·59	9·5	9·5	3	·556	
1345	+44° 4125	20·3	44 48	293·8	2·61	9·3	9·7	2	·743	
1346	+43° 4203	22·5	44 2	311·1	2·99	9·5	11·7	3	·877	AB
				360·7	7·29	C=	13·4	2	·919	AC
1347	+44° 4144	24·9	44 34	150·4	3·57	9·4	13·0	2	·724	
1348	+43° 4279	38·6	44 13	102·4	2·73	8·6	13·5	2	·987	
1349	+44° 4309	57·6	44 19	166·3	4·99	8·8	13·6	4	·800	
1350	+43° 4419	23 9·4	44 5	79·7	7·74	8·9	11·5	3	·873	
									1915.	
1351	+42° 4655	18·2	43 12	141·7	3·92	9·5	10·8	2	·008	
1352	De 26	33·9	43 58	70·7	2·22	9·3	10·5	3	·977	AB
				292·2	4·91	C=	14·8	3	·977	AC
1353	+44° 4488	41·6	44 23	78·8	1·98	9·5	11·2	3	·880	
1354	+44° 4494	43·2	44 39	339·5	6·94	9·5	9·8	2	·897	
1355	+43° 4573	49·9	43 28	11·9	3·61	9·4	14·0	3	·975	
1356	+43° 4596	54·3	44 2	106·9	2·51	9·3	12·0	2	·966	

Notes.

1296. $0^m\ 7^s\cdot4$ *p*, 0′ 41″ *s*, Espin 444.
1310. This star has the same R.A. as B.D. + 44°, 639, but is 1′ further N.
1321. Colours: orange, blue. There are two distant *comites*, 14 mag. P 179°·4, D 23″·84, 2 nts.; 11·5, P 317°·3, D 33″·17, 2 nts.
1324. *Comes*, 13·5, P 144°·5, D 23″.
1334. *N* star of group of four.
1335. In B.D., 9·4 mag. Much underrated. *Cat. Ast. Ges.* (Bonn), 8·5 mag.
1337. *Comes*, 12 mag. P 147°·8, D 22″·5.
1341. A star, 9·4, P 327°·5, D 33″.
1352. The faint *comes* has hitherto escaped detection. No mention is made of it by β, although he measured De 26 on one night with the 18½-in. at the Dearborn Observatory in 1878, and on three nights with the 40-in. at the Yerkes in 1903. It was measured with the 24-in. in moonlight on all three nights.

The Influence of Local Atmospheric Cooling on Astronomical Refraction. By Prof. Sir Joseph Larmor, F.R.S.

1. The following considerations, which can hardly be altogether new, were prompted by reading the note on latitude variation in the Annual Report for 1913 of the R.A.S.* A question is there broached, whether the residual Kimura annual variation in latitude, symmetrical around the Earth's axis and amounting in amplitude to about 0″·06, is of such magnitude as can reasonably be referred to changes of refraction. As the term attains its extreme values at midsummer and midwinter, vanishing at the intermediate equinoxes, such an inquiry is encouraged.

In this connexion we need not trouble about the curvature of the Earth. We can take the atmosphere to be a flat stratified plate of air; and then, as is well known, the determination of the refraction becomes very simple and direct. For if we imagine a thin horizontal layer of vacuum, interrupting the atmosphere just above the level of the observer, a ray will emerge from above into this crevice along a path parallel to its original direction, because it has passed across a flat stratified plate: the ray will thus be incident in its original direction on the lower horizontal surface of the crevice, to be there refracted into its actual direction as presented to the observer, the crevice being, by the principles of wave transmission, without effect on the final direction of propagation.

* *Monthly Notices*, lxxiv. p. 370.

If then z is the true zenith distance of a star, z_0 the apparent, and μ the refractive index at the level of the observer, the refraction r, equal to $z - z_0$, is given by the law of sines

$$\sin z = \mu \sin (z - r),$$

leading, when r is small, to the usual formula applicable to large altitudes,

$$r = (\mu - 1) \tan z.$$

For smaller altitudes a correction for terrestrial curvature must be added.

The value of μ depends on the density of the air at the place, as determined from its pressure and temperature; for changes in each of these data a correction is in practice duly made. It is the density at the position of the object-glass that here enters, the presumption being that the metallic frame of the observing telescope ensures, by its conducting quality, constancy of temperature, for it must ensure absence of flexure from thermal causes throughout its structure.

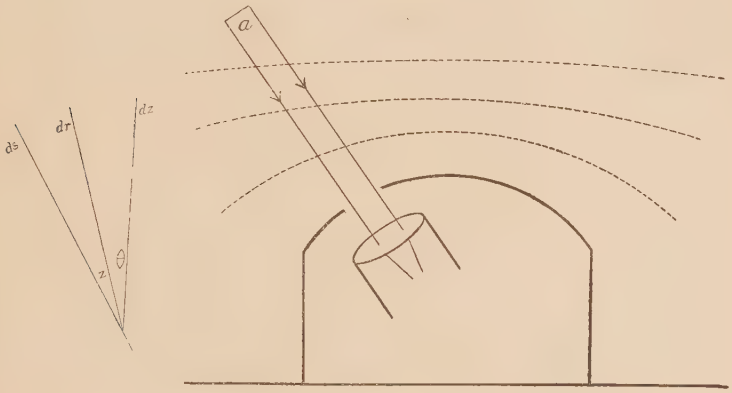
But such considerations apply also to the dome in which the telescope is enclosed; being itself at a temperature more or less uniform, it will disturb the horizontal stratification of the air above it. It is true that in usual circumstances the temperature in the free air falls only about 1° C. in a rise of 300 feet. But the phenomenon of the deposition of dew involves much steeper fall close to the wetted surface. If this surface is dome-shaped instead of flat, the effect is doubtless much mitigated by the sliding of the cooled air down the sloping surface; yet local difference of temperature is still maintained by radiation into open space.

We are thus prompted to examine the effect on the general refraction that is produced by a local elevation of the strata of equal density, elsewhere flat, in passing over the dome, whether such lifting arise from a sharp temperature gradient through a few feet over the dome, or from a more gradual one extending through a greater height.

2. To attain this end, we can establish our imagined horizontal crevice in the atmosphere just above this region of disturbance, say, at a height h from the object-glass, and trace out what happens to the ray in its path beneath that level. The value of the refraction at the level of the crevice is obtained by subtracting from the usual refraction $(\mu - 1) \tan z + \ldots$, calculated for the position of the object-glass, an amount $-\frac{\partial \mu}{\partial h} h \tan z$; viz. we have to add $\frac{\partial \mu}{\partial h} h \tan z$, or say $(\mu_2 - \mu_1) \tan z$. We have then further to add the actual additional refraction due to the ray crossing the curved strata of equal index down to the telescope, as indicated in the diagram. To obtain its value we compare the ray with a parallel ray at very small distance α from it, say below it. As $\Sigma \mu \delta s$ is the

same for all rays between the same two wave-fronts, the difference of its values for these two rays, when divided by a, is equal to the angle the wave-front has been slewed round, and is therefore the additional refraction in this part of the path.

At any point on the ray, let θ be the slope of the stratum of constant index: then as δn, along the normal to this stratum, is in the direction of the gradient of μ, we have, δs being an element



of the ray, as before, measured upwards let us say to agree with δh, and $\delta s'$ at right angles to it,

$$\frac{\partial \mu}{\partial s} = \frac{\partial \mu}{\partial n} \cos(z - \theta), \qquad \frac{\partial \mu}{\partial s'} = \frac{\partial \mu}{\partial n} \cos(z - \theta + \tfrac{1}{2}\pi).$$

The value of μ at the nearest point on the parallel ray being $\mu + \frac{\partial \mu}{\partial s'} a$, is thus $\mu - \frac{\partial \mu}{\partial s} a \tan(z - \theta)$.

The additional refraction due to this part of the path is therefore

$$-\int \frac{\partial \mu}{\partial s} ds \tan(z - \theta),$$

integrated upward, which is equal to $-(\mu_2 - \mu_1)$ multiplied by the mean value of $\tan(z - \theta)$ along the path. To get the whole disturbance due to the building or dome we have to add to this $(\mu_2 - \mu_1) \tan z$, as above, where μ_2 refers to the upper end, and μ_1 to the lower end, of the curved part of the ray.

The effect of the crowding down of the temperature gradient on to the dome, where the strata are oblique, is thus equal to

$$(\mu_2 - \mu_1)\{\tan z - \text{mean } \tan(z - \theta)\}, \qquad . \qquad . \qquad (A)$$

and is usually, as it clearly ought to be, a diminution of the refraction, $\mu_2 - \mu_1$ being negative; it would count as an apparent increase

of zenith distance. It may well be sensibly different in winter from what it is in summer. If it is smaller in summer, the zenith distances as observed are then in defect; for example, the latitude, as determined from a northern star alone, would be in excess. We proceed to attempt an estimate of the possible discrepancy.

It is some moderate fraction of $\mu_2 - \mu_1$, the difference of indices at the ends of this terminal curved part of the ray, say usually of the order of one-fifth, while it may be much higher. If we take the local differences in μ as roughly due to temperature τ alone, measured from absolute zero, then locally $\mu - 1$ is proportional to τ^{-1}, so that

$$\mu_2 - \mu_1 = \delta\mu = -\frac{\mu - 1}{\tau}\delta\tau.$$

Here $\mu - 1$ is about ·0003, that being the value given by the simple formula $r = (\mu - 1)\tan z$, from the knowledge that the refraction is about 58″ $\tan z$, and τ is say 273°. Thus $\delta\mu$ is of the order $10^{-6}\delta\tau$: and an abnormal fall of 1°C. in a layer, whose thickness is of minor importance, surrounding the dome, would diminish the refraction by the order of one-fifth of 0″·2. For example, an effect equal to the whole Kimura latitude-difference 0″·06 between summer and winter would need a local upward change of temperature amounting to two or three degrees centigrade.

But such estimates of local error of refraction as are here found for a single star (here taken north of the zenith) do not apply to latitude determinations made from several stars, some north and some south; for these additional refractions would presumably compensate on the whole, though they would appear if the two groups of observations north and south of the zenith were analysed separately. The case would be different if the strata of the atmosphere were tilted as a whole; but it may be estimated from formula (A) that a horizontal pressure gradient of 1 in n per mile in an atmosphere of homogeneous height H (actually about five miles) would shift the zenith by about 0″·0003 $H/n \cos^2 z$, so that for an error of this type to approach the value 0″·06 the mean horizontal barometric gradient would have to be at least $\frac{1}{40}$ of the total pressure, per mile, which is dynamically a quite impossible amount.

The question whether considerations of barometric gradient could have any actual bearing on the puzzling Kimura seasonal effect on latitude would be put to one kind of test by comparing the values derived from the two hemispheres, where the seasons are opposite. If these values agreed symmetrically in phase, in the sense that the variation of north latitude is positive when the variation of south latitude is positive, then such a suggestion could have no place. As a fact, the observations give a deflection in the same direction in both hemispheres at the same time of year, for they are found to be such as an imagined axial displacement of the Earth's centre of mass would account for. Considera-

tions of local refraction are, however, excluded, unless indeed it be unsymmetrical, as the Talcott procedure involves observations at the same altitude on both sides of the zenith. A tilt in the stratification of the atmosphere, swaying between summer and winter, would serve; but the formula indicates for it, as above, something of the order of several degrees of angle, which is an impossible amount to contemplate as spreading over any considerable distance.

In any case, this general estimate of the possible magnitude of these irregularities in the refraction may not be superfluous. Whether they ever affect meridian observations to any sensible degree is for practical astronomers to decide; indeed, the question can readily be tested by attaching for a time an ordinary recording thermograph at the object-glass of a working transit circle, another a few yards above the dome, and another at some distance on the level.

3. The formula (A) expressing the result of local accidental refraction may be verified directly from the simple consideration that, instead of a ray crossing the atmospheric strata at an angle z, we have one crossing them at a mean angle $z - \theta$. More generally it may readily be shown, by considering a series of differential refractions, that the total deviation of a ray, incident on the strata at angle z, where the index is μ, is equal, for any arc of its path, exactly to

$$\int \tan z \frac{\partial}{\partial s} \log \mu \, . \, ds, \quad \text{or} \quad \int \tan z \, . \, d \log \mu,$$

which is, in fact, a more directly evaluatable form of the well-known formula for deviation $\int \frac{\partial \log \mu}{\partial n} ds$. For example, in the present case, z is nearly constant, say is $z_0 + \delta z$, and the deviation in any arc becomes

$$\int (\tan z_0 + \sec^2 z_0 \, . \, \delta z) d \log \mu,$$

or

$$\tan z_0 \, . \log \frac{\mu_2}{\mu_1} + \sec^2 z_0 \int \delta z \, . \, d \log \mu,$$

where $\log \mu$ may be replaced by $\mu - 1$. If the strata between indices μ_2 and μ_1 are tilted through the same small angle ϵ. then $\delta z = \epsilon_0$, and the effect is

$$\sec^2 z_0 \, . \, \epsilon (\mu_2 - \mu_1),$$

being independent of the thickness of the stratum.

The effect of the curvature of the Earth on the deviation can be determined from the same formula, with the appropriate value of δz, derived from

$$(a + h) \sin (z_0 + \delta z) = a \sin z_0 :$$

but it is to be noted that owing to the changing direction of the vertical, deviation, which is the same as refraction, is not now the same as $\int \delta dz$. In this way Laplace's expression for the refraction for any arc of path is readily obtained.*

4. After the above was written, it was found in looking through Henry Cavendish's extensive MS. calculations † relating to the planning by the Royal Society, between 1772 and 1774, of the Schehallien experiment for the determination of the Earth's mean density, in which Cavendish took a very prominent part, that he had then considered the problem of accidental refraction-error due to the mountain, substantially as here. If the summit is colder than it is at the same level directly above the foot of the mountain, the strata of equal atmospheric density tilt upward towards the summit; and the apparent position of a star, chosen near the zenith to avoid ordinary refraction, will be deflected away from the mountain, thus making the observed effect of attraction of the vertical towards the mountain too small. Cavendish calculated roughly that for a defect of temperature of $12\frac{1}{2}^\circ$ F. at the summit, which he took as an extreme estimate, and for a hill sloping at 21° to the horizon, the error would be about $0''{\cdot}6$ on each side of the hill, depending, as we have recognised, only on the temperature difference and not on the height. The formula (A) gives for a local temperature change of this total amount, from a mountain of that slope to the air above it, a value of about $0''{\cdot}5$ for z small, if the strata are taken parallel to the slope, whereas Cavendish made them inclined to the slope so that the temperature fell along it. In the Schehallien observations the effect of the attraction of the mountain came out as $11''{\cdot}6$, which Cavendish's extreme estimate for refraction would increase by 10 per cent., thus, as it happens, leading to consistency with modern determinations. There does not appear to have been any attempt to apply such a correction, either to Maskelyne's Schehallien observations, or to the later series by Sir Henry James at Arthur's Seat, where the result was 3 or 4 per cent. too small, while owing to flatter conditions the refraction error would be much less: see the account in Poynting's Adams Prize Essay of 1894 on *The Mean Density of the Earth*, pp. 15–22.

* As, in fact, in Lord Rayleigh's important paper "On the Theory of Stellar Scintillation," *Phil. Mag.*, xxvi., 1893: *Scientific Papers*, vol. iv. pp. 66-72, especially p. 71.

† Now under examination with a view to a second completing volume of Collected Papers.

On Irregularities in the Earth's Rotation, in Relation to the Outstanding Discrepancies in the Orbital Motion of the Moon. By Prof. Sir Joseph Larmor, F.R.S.

1. The results of the application of the law of gravitation to the lunar motion have now been summed up in magisterial manner by Prof. E. W. Brown,* as the culmination of his classical investigation on the Lunar Theory. The circumstance that there remains an outstanding irregularity in the orbital motion, composed roughly of a fluctuation of 13 seconds of arc on each side of a mean in a half-period of about 140 years, combined with a like fluctuation of about 3 seconds of arc in the shorter half-period of about 35 years, has been felt to create an intolerable discrepancy, which demands every effort of the gravitational astronomer to resolve. No higher tribute than this could be paid to the extreme refinement and exactness of the gravitational explanation of the celestial motions, a department of knowledge which leaves far behind the highest amounts of precision that the presence of intractable disturbing agencies allows to us in other branches of physical science.

The object of this note is to examine such grounds as may exist for attributing all or part of these discrepancies, whose rate of change amounts to the order of 15 to 20 seconds of arc per century of time, to irregular changes in the Earth's velocity of rotation on its axis, due to rearrangement of surface masses. For such changes would introduce fluctuating uncertainties into the fundamental reckoning of time that would be, in astronomical procedure, thrown on to the celestial motions which that time is employed to specify. This suggestion towards a possible explanation can claim classical authority of long standing. In his address to the Mathematical and Physical Section of the British Association at Glasgow in 1876,† Lord Kelvin referred, towards the end, to these fluctuations of the lunar motion, of which he had just received private information from Newcomb; and he agreed with Newcomb in ascribing them to the cause here described. The subject of the diminution of the Earth's axial motion by tidal friction had occupied his attention before 1867.‡

The actual astronomical evidence that has been brought forward is of two types. First, the times of day are recorded at

* *British Association Report*, Australia, Sept. 1914, "Address on Cosmical Physics."

† Reprinted in *Popular Lectures and Addresses*, vol. ii.; see p. 271. This explanation is developed numerically in 1888 in a later paper on Polar Ice-caps, *loc. cit.*, p. 319; while the effect of tidal friction in slowing the Earth's rotation was estimated in 1868, in a memoir on Geological Time, *loc. cit.*, p. 10.

‡ Cf. Thomson and Tait, *Nat. Phil.*, ed. i., 1867, § 276; also § 830, where an estimate of the ratio of this effect to its rotational reaction on the body producing the tides is quoted from a private communication made by J. C. Adams, which will be referred to and confirmed later.

which some ancient eclipses occurred: this will be affected the same way whether there be a retardation of the Earth's motion of rotation, or an equal acceleration of the Moon's motion, causing it to gain faster on the Sun. Second, observations during modern times have revealed the irregularities, above stated, in the Moon's position among the fixed stars, which prove to be the same whether that position is determined directly by occultations, or indirectly by meridian transits with reference to a clock which is itself rated by transits of fixed stars; in either case, if the Earth has been rotating too slow, the actual position of the Moon will be referred to a time behind the actual absolute time, involving apparent acceleration of its motion.

Such an irregularity in the practical measure of time, viz. the Earth's rotation, as is here contemplated, would be a universal cause, introducing similar discrepancies into all periodic phenomena in the solar system. Although there is no other feature sensitive enough to form a definite test of so small an effect, there are, it appears, two in which the fluctuations might just show, namely, the motions in longitude of the Earth and Mercury. The very interesting comparative diagrams printed near the end of Prof. Brown's recent address (*loc. cit.*) possibly show some trace of correlation of this kind: at any rate, we may take it that they contain all that can be made out in this regard.

It is proposed here to estimate what possibilities exist in this field. It will appear that there ought to be a tidal secular apparent acceleration of the Moon's motion, which may conceivably be of about the order usually thought to be outstanding, and that the fluctuating irregularities which remain unexplained are of an order not inconsistent with their being due, in substantial part, to terrestrial movement of material.

2. Irregular changes in the Earth's velocity of rotation are to be ascribed to transfers of terrestrial material, of accidental, for example meteorological, character. Merely local transfer, for example disturbance due to an earthquake, will be practically ineffective. To alter sensibly the moment of inertia of the Earth the transfer must be from places near the axis of rotation to places remote from it, or conversely. Such change can occur through displacement of the oceanic surface waters. But nothing except local movements can be expected inside the solid surface of the Earth, either from earthquakes or other causes: the unchangeableness of the solid Earth, as a whole, is one of the remarkable results of the astronomical data.

For example, the necessary conditions are satisfied by water melting off from an elevated antarctic ice-cap and spreading uniformly over the whole ocean surface. This case was first discussed by Dr. J. Croll in connection with the origin of changes of sea-level and of climate: and it has been considered at length under the present aspect by Lord Kelvin in 1888.*

* *Loc. cit., ante.*

The necessary conditions are also satisfied by a sudden or gradual local rise or subsidence of an ocean-floor; for this must cause compensating rearrangement of water over the whole ocean. According to the late Dr. J. Milne, the effect of an earthquake may be a displacement over a large area, vertically or horizontally, through as much as 10 feet. Such a fall of the sea bottom, 10 feet, say over 400 miles square, if it is balanced by the rise of a land surface and not by an adjacent rise of the ocean floor, would lower the level all over the ocean by about $\frac{1}{7}$ of an inch, supposing the ocean to cover $\frac{2}{3}$ of the Earth's surface. If it occur in latitude λ, and the mass of water thus displaced is M, it will alter its angular momentum from $\omega M a^2 \cos^2 \lambda$ to $\frac{2}{5}\omega M a^2$. With the figures of our illustration, this event would diminish the period of the Earth's rotation, as the total angular momentum must remain constant, by the fraction $8 \cdot 10^{-10}$ $(\frac{2}{3} - \cos^2 \lambda)$ of itself, taking its mean density to be $5\frac{1}{2}$; this amounts to $25 \cdot 10^{-3}$ $(\frac{2}{3} - \cos^2 \lambda)$ seconds of time in a year, or at the rate of about $2\frac{1}{2}$ $(\frac{2}{3} - \cos^2 \lambda)$ seconds in a century. If all this discrepancy in the time-standard were thrown on to the Moon's motion, an apparent disturbance at a constant rate of 20″ of arc per century in the Moon's position, which is equivalent to 37 seconds of time, would require a rise or fall of the ocean floor of 420 feet over 400 miles square, if it is near the equator, or half that amount if it is in the polar regions.* The general ocean level would thereby be changed by 6 inches, or by half that amount, in the two cases respectively.† This, or a proportionally smaller rise or fall of the ocean bed over a larger area, thus seems too large an amount to postulate in the course of a century; though tracts in the Pacific near the western coast of South America have been known to drop by 1000 feet.‡ But the present type of explanation of the outstanding lunar irregularities seems to be at any rate partially adequate.

Lord Kelvin verified, at an early stage of his consideration of these subjects, that the Earth's gradual shrinkage, owing to its known rate of loss of heat, is far too small to produce any appreciable effect of this kind. It may readily be verified that the slight wanderings of the Pole cannot be sensibly effective. The most remarkable physical alteration that is going on in the Earth is perhaps the rapid change of its magnetic moment: but the change of dimensions thus brought about, owing to magnetic change of form, is also far too small to be sensible in this way.

* *Cf.* in E. H. Hills and J. Larmor, "The Irregular Movement of the Earth's Axis of Rotation," *Monthly Notices, R.A.S.*, Nov. 1900, p. 26; there too it is shown that a displacement of the order in the text must be distributed in longitude if it existed, for if localised it would affect latitudes by 2″, which is far too much.

† The quicker fluctuation by itself would even demand a rise and fall of this order about every seventy years, which is rather beyond probability.

‡ *Cf. Proc. Roy. Soc.*, Dec. 1908, p. 95, "On the Relation between the Earth's Free Precessional Rotation and the Resistance to Tidal Deformation." [The large drop above mentioned is, I am informed, now considered doubtful.]

3. The lunar torque on a spheroidal tidal excess of water, of maximum height h above mean level and angular lag θ behind the moon, is, when referred to the principal axes of the spheroid,

$$G = 2\frac{M}{R^3}\Sigma\delta m(x\cos\theta - y\sin\theta)(x\sin\theta + y\cos\theta).$$

$$= 2\frac{M}{R^3}\tfrac{1}{2}\sin 2\theta \cdot \tfrac{2}{11}E \cdot \tfrac{3}{5}ah,$$

by using the known values of the moments of inertia of a spheroid of water. This produces a retardation of the Earth's rotation given by

$$-\frac{d\omega}{dt} = \frac{G}{\frac{1}{3}a^2E} = \frac{M}{3R^3}\frac{h}{a}\sin 2\theta$$

where

$$\frac{M}{R^3} = \frac{1}{82 \cdot 60^3 a}g, \quad \text{and} \quad g = 32$$

in foot, pound, second units.

The loss in absolute angular measure in a century, due to this accelerating cause, is

$$-\tfrac{1}{2}\frac{d\omega}{dt}(100 \cdot 365 \cdot 24 \cdot 60 \cdot 60)^2,$$

which is convertible into seconds of time by the factor 13700. On reckoning out, it goes slow in a century by $91\,h \sin 2\theta$ seconds, when h is in feet, assuming the ocean to cover the whole surface: this rough estimate comparing apparently with Lord Kelvin's $137h \sin 2\theta$.*

If the attraction of the tidal water could be neglected, the value of h for the ideal lunar equilibrium tide would be given by $g \cdot \frac{3}{2}h = \frac{3}{2}\frac{Ma^2}{R^3}$, by equating the opposite changes of potential, where $M = \frac{1}{81}E$, $R = 60a$, $\frac{E}{a^2} = g$; this would make h equal to about $\frac{7}{6}$ feet. The augmentation due to self-attraction of the tide is small.†

4. The amount of its lag θ depends on the retarding friction that is in operation. For the actual tide, to which, however, this equilibrium theory is not applicable, except most roughly, some light on its value is thrown by a remark of Sir George Airy, which has been elaborated by Lord Kelvin, namely, that spring tides occur more than 12 hours after the conjunction of Sun and Moon.‡

If we might assume that the tidal displacement is that of a

* Rede Lecture, Cambridge, May 23, 1866, "On the Dissipation of Energy"; *Popular Lectures*, vol. ii. p. 65, where the calculation is made differently for a special numerical example. The value given, for a viscous liquid spheroid, by Sir G. H. Darwin (Thomson and Tait, ed. ii., Appendix G, *a*) is difficult to compare.

† *Loc. cit.*, *Popular Lectures*, vol. ii. p. 26.

‡ *Cf.* Thomson and Tait, §§ 804, 817.

fundamental free oscillation of the ocean, its co-ordinate ϕ would be given by an equation of the form

$$\frac{d^2\phi}{dt^2} + \kappa\frac{d\phi}{dt} + n^2\phi = \mathrm{F}\cos pt,$$

where, when the friction is not prominent and κ is therefore small, the period of the transient free oscillation is $2\pi/n$ and the period of the forced permanent tide is $2\pi/p$. This tide is expressed by the particular solution

$$\phi = \frac{\mathrm{F}\cos(pt+\alpha)}{\{(n^2-p^2)^2+\kappa^2p^2\}^{\frac{1}{2}}},$$

where

$$\tan\alpha = \frac{\kappa p}{n^2-p^2}.$$

Thus, neglecting κ^2, the tide is *reversed*, so that low water is under the Moon, and the phase is *retarded*, if $n<p$, *i.e.*, if the period of free oscillation exceeds the tidal period, as is actually the case; and the lunar tide, being nearer the free period, would be magnified by the sympathy of periods.* As κ is the same for both solar and lunar periods, the lag, which is α/p in time, is inversely as n^2-p^2, and increases as the periods approach, up to a limit near which this reasoning becomes inapplicable.

Observation, as above quoted, shows that the difference of the lags for the lunar and solar tides exceeds 6°. If we thus took, as a fair figure, 10° as the lag for one of them, and took h to be $\frac{7}{6}$ of a foot as above, the lag in time due to tidal friction in a century, as reckoned above, would amount to 36 seconds, of which $\frac{1}{2}$ (see § 5), giving 10″ of arc, would appear as advance in the Moon's position. This, as it happens, is rather more than the value which has been assigned to the unexplained acceleration of the Moon's motion on the basis of Airy's interpretation of ancient eclipses.† Approximate coincidence would be an accident: but the order of magnitude is evidence that, so far from tidal friction not being an effective *vera causa* for an apparent secular acceleration of the motion of the Moon, it seems to be competent to account for an effect of the same order as the actual amount which has been assumed to be outstanding.

If we knew from observation the actual distribution of the tide over the Earth, the actual torque could be calculated with safety by quadratures, and a definite result obtained. Perhaps even with modern tidal knowledge this would not be feasible: but it ought not to be difficult from general inspection of the co-tidal lines on the charts to recognise in a general way whether the retarding torque on the Earth, and consequent permanent lunar

* The other case, tide direct and phase accelerated, of course also places full tide after the Moon has passed the meridian, and so gives also a torque in retarding direction on the Earth.

† *Cf.* Thomson and Tait, part ii. § 830, where the estimate is 6″, plus 1½″ on account of thermodynamic atmospheric acceleration. [But the 10″ in the text must be reduced to 7″: see next footnote.]

apparent accleration, is actually much smaller than the value (itself barely sufficient) * here deduced from an equilibrium theory.†

5. In this analysis of tidal friction we have tried to estimate the retarding influence on the Earth. In addition to its influence in causing lunar apparent orbital acceleration by delaying the time, there is still the reaction on the true motion of the Moon to be considered. For simplicity in estimating its order of magnitude, imagine an Earth-Moon system isolated, the two bodies rotating about parallel axes so that the equator and ecliptic coincide. The angular momentum of the system will be conserved, as there is no dynamical interference from outside, though there may be thermal. Thus if a, E, ω are the radius, mass, and angular velocity of the Earth on its axis, a', M, ω' the corresponding quantities for the Moon, Ω its orbital angular velocity, R its orbital radius with respect to the centre of mass of the Earth and Moon, or say with respect to the centre of the Earth, as we can imagine that point to be fixed and take moments round it, without excessive error, then,

$$\mathrm{E}k^2\omega + \mathrm{M}k'^2\omega' + \mathrm{MR}^2\Omega = \text{constant},$$

where, assuming the Laplacian law of internal density, $k^2 = \frac{1}{5}a^2$. In this equation of conservation, the coefficients of ω, ω', and Ω are roughly as 1, $\frac{1}{1000}$, 140. As ω' is equal to Ω we see that, in a rough estimate of exchanges of angular momentum, the second term will not sensibly influence matters. Hence simultaneous changes in ω and Ω are about as -140 to 1 absolutely, or relative to the lengths of the month and day they are as about -5 to 1.

This estimate is, however, on the supposition that the moments of inertia remain constant, which requires that R be kept constant by constraint. Actually, diminished velocity of the Moon must increase R, in order that the centrifugal reaction may continue to balance the gravitational attraction, viz. in order to maintain the relation

$$\Omega^2\mathrm{R} = (\mathrm{E} + \mathrm{M})/\mathrm{R}^2.$$

Thus $\Omega^2\mathrm{R}^3$ remains constant: therefore

$$\delta(\mathrm{MR}^2\Omega) \propto \delta\Omega^{-\frac{1}{3}}, \text{ and so } = -\tfrac{1}{3}\mathrm{MR}^2\delta\Omega,$$

so that the effective relative coefficient of Ω is not -140, but $\frac{1}{3}$ of that number with sign changed, viz. $+47$; and the proportional effects on the month and the day are about as 5 to 3, both in the direction of increase.

* Too small by $\frac{1}{3}$ when the continents are allowed for.

† Prof. Eddington points out to me that the tide in the solid Earth ought also to come in. If, as seems very likely, that tide is almost purely elastic, it can contribute nothing to the retardation. This solid tide is $\frac{1}{3}$ of the total ideal equilibrium tide (*cf. Proc. Roy. Soc.*, Dec. 1908, p. 94): thus $\frac{1}{3}$ of the moment estimated in the text must be deducted from the result of the quadratures, which is a very substantial reduction.

To estimate how the frictional loss of energy is divided between them we note that $\delta(\frac{1}{2}MR^2\Omega^2) \propto \delta\Omega^{\frac{2}{3}}$, and so $= \frac{2}{3}\frac{1}{2}MR^2\Omega\delta\Omega$. Thus the ratio of the changes of energy, both losses, is that of the changes of angular momentum, which is unity, multiplied by the ratio of ω to 2Ω, so that the Earth's rotation sustains $\frac{14}{15}$ of the whole loss of energy.

But hitherto we have been considering only mutual action between Earth and Moon, and have neglected to take in the change of ω due to the solar tide. The solar tide-generating force is roughly $\frac{2}{5}$ of the lunar, hence the solar torque is roughly $(\frac{2}{5})^2$ of the lunar. Thus in all, we have instead of 47 a ratio $(1 + \frac{4}{25})$ 47, or 54. Hence an accumulated lag, due to tides, of 1″ of arc on the Moon's true position among the stars corresponds to a deficiency of 54″ of arc or $3\frac{3}{5}$ seconds of time in the Earth's rotational position,* so that it has gone slow by this number of seconds, making the Moon appear to be nearly 2″ in advance. The total apparent advance of the Moon's position is thus $\frac{1}{2}$ of that due to lag of time alone.†

This agrees with the result communicated by J. C. Adams for the first edition of Thomson and Tait (1867, § 830). His argument, which doubtless followed these lines, does not seem to have ever been published. In Thomson and Tait, ed. ii. (Appendix G, *a*), Sir George Darwin, from the special numerical data for the problem of viscous liquid spheroids calculated in his paper, *Phil. Trans.*, 1879, verified that Adams' ratio held good.

In 1874 the same method of momentum was employed by J. Purser (*Brit. Assoc. Report*), much as above, to answer Airy's unsolved query as to how the loss of energy is divided between the Earth and Moon; and in 1879 the method gave, by graphical treatment in Sir George Darwin's hands, his famous analysis of the past history of the Earth-Moon system (see Thomson and Tait, ed. ii., Appendix G, *b*).

Added Jan. 28.—The maximum estimate in § 4 *supra* for the effect of tidal friction is an apparent *advance* of 10″ of arc in a century of the Moon's motion, *i.e.* an *acceleration* of twice that amount. But this value must be reduced by about one-third (see footnote), on account of the tide being in part an elastic Earth-tide; this correction, as it happens, brings it close to the amount usually assumed as outstanding. There would also be involved an apparent advance of the Sun's motion among the stars, amounting to $\frac{1}{13}$ of $2 \times 7''$ in a century, such as Prof. Brown has tried to find traces of.

Prof. Eddington has drawn my attention to a remarkable synthesis of these matters, avowedly tentative and apparently independent of the discussions of previous writers, by Dr. Cowell (*Monthly Notices*, vol. lxvi. p. 354), and has assisted me to under-

* The neglected obliquity of the ecliptic will diminish this estimate by something like $\frac{1}{13}$ of itself, an amount in other ways uncertain.

† The necessity for this factor was pointed out to me by Prof. Eddington.

stand the exposition, which I am not alone in having found difficult. My paraphrase and criticism of the argument, which, if correct, may be of use to others who do not think astronomically, is as follows.

From his discussion of ancient eclipses, demurred to, however, by Newcomb, Dr. Cowell feels compelled to ascribe an outstanding apparent advance of 5″ in a century to the Moon's motion and of 4″ in a century to the Sun's motion among the stars, or accelerations of 10″ and 8″ respectively. If these effects are both ascribed to slowing of the Earth's rotation due to tidal action, the latter figure will give its amount, for there is no other source of solar acceleration: * there would have to be a delay of $365 \times 4''$ of arc in the Earth's rotation, so that it would go slow by about 100 seconds of time in a century. This is more than four times the extreme higher limit found for possible tidal effect in § 4 *supra*, as corrected in this note; and, as Dr. Cowell remarks, it is greatly in excess of current estimates. As he also points out, an apparent sidereal advance of the solar motion, of 4″ of arc in a century, would be too great for the *Nautical Almanac* to ignore, though the value $2 \times \frac{7}{13}''$, or say 1″, *supra* might perhaps not be so.

If, however, Dr. Cowell's value is for the moment adopted, the conservation of angular momentum in the Earth-Moon system will determine the concomitant effect on the Moon's motion. Dr. Cowell refines on previous estimates, such as the one discussed *supra*, § 5, by including the eccentricity e of the lunar orbit. The equation of angular momentum, neglecting as before that of the Moon's rotation about its axis, now takes the form, inclination being neglected for the present rough purposes, R being now mean distance so that n^2R^2 is constant,

$$Ek^2N + MR^2n(1 - e^2)^{\frac{1}{2}} = \text{constant},$$

where N, n are the mean motions of terrestrial rotation and lunar orbit (represented *supra* by ω, Ω), so that for example the Moon's longitude is $\int ndt + \ldots$.† If we take for k^2 the Laplacian value $\frac{1}{3}a^2$ instead of Dr. Cowell's rougher $\frac{2}{5}a^2$, the equation of variation is

$$\frac{\delta n}{n} - \frac{23}{36}\frac{\delta N}{N} + \frac{e}{1 - e^2}\delta e = 0.$$

For the Earth's rotation $\delta N/N$ is -6.10^{-8}, as required by the assumed solar acceleration. If the term in δe is neglected this would give for $\delta n/n$ the value -4.10^{-8}, and for the Moon's advance, of tidal origin, viz. a real part due to tidal dynamical reaction and

* Tidal influence is entirely negligible.

† That is, $\int\left(n_0 + \frac{dn}{dt}t + \ldots\right)dt + \ldots$, so that the *advance* in unit time due to a constant part f in the *acceleration* dn/dt is $\frac{1}{2}f$. The astronomical usage, which Dr. Cowell follows, is to give to this advance the name secular acceleration.

an apparent part owing to the lag of astronomical time behind absolute dynamical time, one-half of $\delta n - \frac{n}{N}\delta N$, which is $1.10^{-8}n$ or $16''$ of arc in a century, instead of 3.10^{-9} or $5''$ of arc. This excessive amount Dr. Cowell proposes to reduce towards the latter figure, which his view requires, by aid of the term involving δe: complete identity would demand, with the Laplacian value of k^2, for the change of e per century the value 180.10^{-9} as against Dr. Cowell's estimate 54.10^{-9}.

Apparently the legitimacy of this course is amenable to test. We have to compare the magnitudes of the disturbances $\delta n/n$ and $e\delta e$, terms in the equation of variation, which arise in the lunar orbit from the retarding attraction of the tide. As the orbit is not far from circular, we may employ the known equations * giving the effect of a tangential retarding force T, viz. r being focal distance and a mean distance in an elliptic orbit,

$$\frac{1}{a}\frac{da}{dt} = -\frac{2}{3}\frac{1}{n}\frac{dn}{dt} = -\frac{2a}{\mu}\mathrm{T}v$$

$$\frac{1}{e}\frac{de}{dt} = -2\frac{1-e^2}{e^2}\left(\frac{a}{r}-1\right)\frac{\mathrm{T}}{v},$$

where

$$v^2 = \mu\left(\frac{2}{r}-\frac{1}{a}\right).$$

It appears from them that $e\delta e$ is to $\delta n/n$ in the ratio

$$\frac{2}{3}\frac{n^2a^2}{v^2}(1-e^2)\left(\frac{a}{r}-1\right), \text{ or roughly } \frac{2}{3}\left(\frac{a}{r}-1\right).$$

The time-average of $a/r - 1$ round an elliptic orbit works out to be zero. Thus for an orbit not far from circular, tidal change in eccentricity seems not to affect sensibly at all the orbital acceleration.

* Adapted from Cheyne, *Planetary Theory*, §§ 120–1. as the source nearest at hand.